

# Attack and defense

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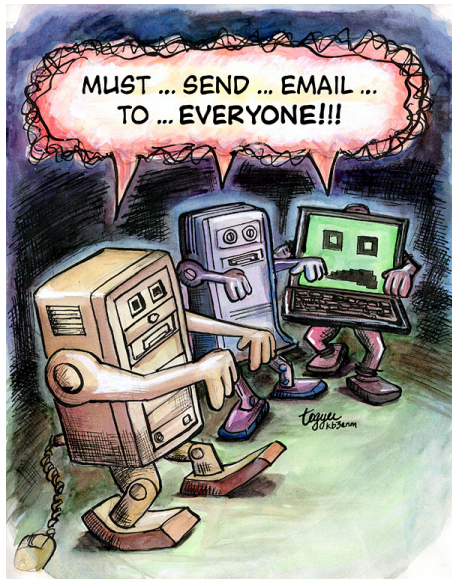
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# This talk



<http://uvmzombies.blogspot.com.au/2013/02/computer-zombies.html>

# Botnets

- Sophisticated distributed systems comprising millions of computers with decentralized control.
  - Network of “zombie” computers infected with malicious programs (“malware”) that allows criminals (“botnet herders”) to control the infected machines remotely without the users’ knowledge.
- Used to
  - ▶ execute Distributed Denial of Service (DDoS) attacks.
  - ▶ harvest credit card information, personal data, financial information, email passwords, etc.
  - ▶ carry out phishing attacks, send out spam, carry out search engine spam, install adware, engage in click fraud.
- Sometimes they are leased out to others, who use them for the above causes.
- If you have a pulse, you’re a target. Anybody’s information has a value.
- Any, even “non-sensitive”, information is valuable. Names, addresses, contacts can be monetized, e.g., sold for social phishing attacks.

# Botnets

- There is a well-organized industry behind this with advertised prices for both outputs (e.g., credit card information) and inputs (e.g., malware-as-a-service).
  - ▶ Prices that depend on quality.
  - ▶ Try-before-you-buy offers.
  - ▶ Bulk offers.
  - ▶ “Google Analytics” for the bad guys, etc.
- Some organizations behind this are really big.

## Example (Rock Phish)

- ▶ High-tech phishing. Practically undetectable & unblacklistable.
- ▶ Huge: Peter Gutmann (UoA) estimates US\$0.5 – US\$1B/year revenue.
- ▶ Scary: Joseph Menn writes about Rock Phish as organized crime, including kidnapping of anti-crime investigator’s daughter.

# Protection

- Some targeted at large institutions: Companies offer banks and other organizations likely to suffer from phishing attacks round-the-clock services to monitor, analyze, assist in shutting down phishing websites, or to implement two-factor authorization, which is being used increasingly.
- Some targeted at end-users: Spam filters target phishing email, firewalls, switches, routers.
- Properties of protection
  - ▶ It is privately costly to invest in protection.
  - ▶ There are positive externalities from investing in protection.
  - ▶ It affects the optimal choice of attackers.

# Biological attacks

- Some features of malware attacks are present in biological attacks:
  - ▶ Contagion. Possibility to protect. Externalities of protection. Indirect effects through choices of attackers.

# The project

- Try to understand more of the Economics underlying the malware economy, including the impact of market power.
- Build stylized models of attack and defense with heterogenous populations of defenders and attackers.

# Model



# Model

## Populations

- Continuum of attackers in population  $I$ , mass  $\mu > 0$ . Choose whether to attack.
- Continuum of defenders in population  $J$ , unit mass. Decide whether to pay for protection against attacks or risk suffering loss from attack.
- Attack is successful if and only if the defender did not pay for protection.
- Attackers cannot observe whether defender has protection.

# Attackers

- Attacker  $i$  obtains payoff of  $x_i$  from a successful **direct** attack.
- $x_i$  is continuously, atomless distributed, CDF  $F_X$ ,  $F_X(+\infty) = 1$ .
- Attacker  $i$  also obtains payoff of  $x_i$  from **indirect** attacks on all unprotected defenders his target is connected to during the attack.
- Abstract from exact process for now.

# Attackers

- Utility of not attacking is

$$U_i(\text{no attack}) = 0.$$

- Let mass of defenders not taking protection be  $\lambda \in [0, 1]$ .
- Model expected utility of attacking as

$$U_i(\text{attack}) = \alpha(\lambda)x_i + \beta(\lambda),$$

- $\alpha(\lambda)$  positive and increasing;  $-\beta(\lambda)$  positive and increasing.

# Attackers

- Attack if

$$U_i(\text{attack}) = \alpha(\lambda)x_i + \beta(\lambda) > 0 = U_i(\text{no attack})$$

or

$$x_i > \frac{-\beta(\lambda)}{\alpha(\lambda)}.$$

- Increase in  $\lambda$  means fewer protected defenders, should make attack more profitable

$$\frac{d}{d\lambda} \left[ \frac{-\beta(\lambda)}{\alpha(\lambda)} \right] \leq 0.$$

- Proportion of attackers choosing not to attack:

$$\chi = F_X \left( \frac{-\beta(\lambda)}{\alpha(\lambda)} \right).$$

# Defenders

- Defenders have a choice between cost of protection and the chance of suffering a loss.
- Denote the loss if she is directly attacked and does not have protection by  $S_j > 0$ .
- $S_j$  is continuously, atomless distributed, CDF  $F_S$ ,  $F_S(+\infty) = 1$ .
- Cost of protection  $c(\chi) > 0$  with  $c'(\chi) \leq 0$ .

# Defenders

- Utility if invested in protection

$$V_j(\text{protection}) = -c(\chi).$$

- Fraction and mass of attackers that choose attack:  $1 - \chi$  and  $\mu(1 - \chi)$ .
- Attackers do not target; there may be indirect attacks; abstract from exact process for now.
- Expected utility of an unprotected defender

$$V_j(\text{no protection}) = \delta(\chi)(-S_j).$$

- $\delta(\chi)$  is positive, decreasing, with  $\delta(1) = 0$  and  $\forall \chi \neq 1, \delta(\chi) \in ]0, 1]$ .

# Defenders

- Invest in protection if

$$V_j(\text{protection}) = -c(\chi) > \delta(\chi)(-S_j) = V_j(\text{no protection})$$

or

$$S_j > \frac{c(\chi)}{\delta(\chi)}.$$

- Mass of unprotected defenders:

$$\lambda = F_L \left( \frac{c(\chi)}{\delta(\chi)} \right).$$

# Equilibrium self-protection



# Equilibrium self-protection

## Proposition

Suppose that  $c(\chi) > 0$ , for every  $\chi$ . Suppose that  $\beta(\lambda) \neq 0$  for all  $\lambda$ ,  $\alpha(0) = 0$  and  $\alpha(\lambda) > 0$  for all  $\lambda > 0$ . Suppose  $F_X\left(\frac{-\beta(1)}{\alpha(1)}\right) < 1$ . Then, the game has a unique Nash equilibrium such that  $0 < \lambda^* < 1$  and  $0 \leq \chi^* < 1$ . Moreover:

$$\chi^* = F_X\left(\frac{-\beta(\lambda^*)}{\alpha(\lambda^*)}\right),$$

$$\lambda^* = F_S\left(\frac{c(\chi^*)}{\delta(\chi^*)}\right).$$

- $\alpha(0) = 0$  means attackers cannot gain anything from attacking if all defenders are protected.
- $F_X\left(\frac{-\beta(1)}{\alpha(1)}\right) < 1$  means if no defenders protect, then there must be some active attackers.

# Equilibrium self-protection

- $\alpha(0) = 0$  would not apply if protection was not perfect. Indeed, our formulation allows for less than full protection. Then  $c$  would combine the price paid for partial protection with the expected damage from successful attacks.
- Denote  $\epsilon_c = \frac{dc}{d\chi} \frac{\chi}{c}$  and  $\epsilon_\delta = \frac{d\delta}{d\chi} \frac{\chi}{\delta}$ .

## Proposition

*Suppose that  $c(\chi) > 0$  for all  $\chi$ , and  $\alpha(\lambda) \neq 0$  and  $\beta(\lambda) \neq 0$  for all  $\lambda$ . Suppose  $F_X \left( \frac{-\beta(1)}{\alpha(1)} \right) < 1$ . Then, the game has a Nash equilibrium. This Nash equilibrium is unique if  $\epsilon_c \geq \epsilon_\delta$ , for all  $\chi$ . In this equilibrium, a proportion  $\chi^*$  of attackers do not attack and a proportion  $\lambda^*$  of defenders do not pay for protection (as defined above). This equilibrium is such that  $0 < \lambda^* < 1$  and  $0 \leq \chi^* < 1$ .*

# Equilibrium self-protection

## Inefficiency of equilibrium self-protection

- Marginal defender's choice to invest in protection lowers the mass of active attackers.
- Positive externality onto other unprotected defenders.
- If  $dc/d\chi < 0$  also positive externality onto other protected defenders.

# Market for protection

# Market for protection

- Assume protection is sold at a price  $p$ .
- Given the above propositions, for any  $p$ , there exists a unique equilibrium with

$$\chi^* = F_X \left( \frac{-\beta(\lambda^*)}{\alpha(\lambda^*)} \right),$$

$$\lambda^* = F_S \left( \frac{p}{\delta(\chi^*)} \right).$$

- Demand for protection:  $D(p) = 1 - \lambda^*$ . Under reasonable assumptions on  $\delta(\chi)$  and  $\alpha(\lambda)$ ,  $D(p)$  and  $\chi^*$  are decreasing in  $p$ .

# Market for protection

- Assume price-taking firms that provide protection to a measure  $q$  of defenders incur a cost  $C(q, \chi^*) = qc(\chi^*)$ .
- Then, the allocation in the competitive equilibrium coincides with that in self-protection.
- There is too little protection.

## Welfare loss with market power

- A monopolist would incur  $C(1 - \lambda^*, \chi^*) = (1 - \lambda^*)c(\chi^*)$ .
- Price decrease has two effects

$$\frac{d}{dp} [(1 - \lambda^*)c(\chi^*; \omega)] = \underbrace{(1 - \lambda^*) \frac{d}{d\chi} [c(\chi^*)] \frac{d}{dp} [\chi^*]}_{\text{indirect effect, } >0} - \underbrace{c(\chi^*) \frac{d}{dp} [\lambda^*]}_{\text{direct effect, } <0}.$$

- Monopoly solution satisfies

$$-\frac{1}{\varepsilon_\lambda} = \frac{p - \left\{ c(\chi^*) - (1 - \lambda^*) \frac{d}{d\chi} [c(\chi^*)] f_X(\cdot) \frac{d}{d\lambda} \left[ \frac{-\beta(\lambda^*)}{\alpha(\lambda^*)} \right] \right\}}{p}.$$

- (1) Welfare loss from externality onto unprotected defenders is compounded by monopoly mark-up.
- (2) Monopolist internalizes part of the externality: A decrease in the price increases demand and decreases attacks, making protection cheaper.

# To do

- A lot.
- Networked defenders and attackers that can distinguish defenders with different connectivity.
  - ▶ Who will be attacked?
  - ▶ Who will be protected?
  - ▶ Relative size of externalities?
  - ▶ Pricing of protection?
  - ▶ Cross subsidies between defender groups?
- Organised attacker that hires subset of attackers at a price for a DDoS attack.
  - ▶ How does price for botnet rental depends on decentralized equilibrium?
  - ▶ How does it change with connectivity and the implied change in protection levels?
  - ▶ ...