LEARNING, ENTRY AND COMPETITION WITH UNCERTAIN COMMON ENTRY COSTS

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ABSTRACT. We model strategic market entry in the presence of uncertain, common market entry costs. Two firms receive costless signals about the cost of a new project and decide when to invest. We characterize the equilibrium of the investment timing game when signals are private. We show that competition leads the two firms to invest too early, whereas the urge to thwart information transmission to avoid triggering entry by rivals leads them to delay their investment. Hence, both excess momentum and excess delay can be observed in equilibrium.
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1. INTRODUCTION

When firms decide to develop a product for a new market, they often wait for some signal that says the time is right to enter. During the waiting phase they try to assess questions, like "Is the technology right? Are consumers receptive? How expensive is it to make consumers receptive?" The answers to these questions establish how costly their marketing campaigns will be (and also how successful) and, finally, whether market entry is profitable. To determine the answers to these questions,

firms regularly spend considerable time and resources doing market research to assess the demand as well as various distribution and production alternatives before deciding whether to enter. However, because the right time to enter a market is similar for rival firms, they also attempt to infer the relevant information from their opponents' decisions. Indeed, sometimes firms also seem to be waiting for someone else to make the decision to invest in entry, only to follow suit immediately. Because this copy-cat behavior reduces profits, firms may have an incentive to reduce the informativeness of their entry decisions. They may, for example, pre-commit to randomize over entry at particular dates, such as trade fairs, if they have not yet learned about the entry conditions until then. In such an environment, firms with good news about the entry conditions have an incentive wait until that date to reduce information flow to their rivals. As a second effect, a competitor's decision to follow suit lowers expected profits from entry, which impacts on the decision to gather information about demand and distribution and production alternatives as well as about the timing of entry once all information has been gathered.

In this paper, we study a game of strategic market entry in the presence of uncertain, common market entry costs with the aim to better understand the interplay between experimentation with the goal of acquiring knowledge about the cost of entry and the incentive to prevent information transmission to potential rivals. It is the incentive to prevent information transmission with the goal of avoiding competition in the market that differentiates our model from previous studies, in which investors are waiting for each other to invest, signaling for example the end of a recession.[1]

Introducing post-entry competition we find that the strategic delay and investment failure that is found in these studies is counteracted and may even be outweighed by the urge to beat the competitor to the market and to do so without giving up the information that would induce their entry.

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[1]Previous studies on information revelation and strategic delay have largely abstracted from post-entry competition, see Chamley and Gale (1994), Décamps and Mariotti (2004), and Lambrecht and Perraudin (2003). These studies find that firms delay investment strategically to benefit from information externalities.

To this end, we study investment decisions by two firms which compete to enter a new market. Ex-ante, the firms are uncertain about the entry costs, which we assume to be common, that is, perfectly correlated for the two firms. Firms can gradually acquire signals about the entry cost through market research and experimentation. They can also enter without experimentation. Within this framework, firms face an optimal timing problem. They need to decide when to enter. We assume that entry by two firms dissipates rents: If there is one firm in the market, that firm collects monopoly profits whereas if there are two, they collect duopoly profits. Because we are interested in new markets (as opposed to mature ones), we suppose that they justify entry of two firms only if the entry cost is low and no firm if the entry cost is high. We assume that entry by an uninformed firm is profitable (in expectation) if that firm is a monopoly but unprofitable if it has to share the market with a competitor. We derive the cooperative outcome where firms choose their entry to maximize joint profits and three types of equilibria in the non-cooperative game played by the two firms when signals are their private information.

Our first finding is that there are many equilibria, including those in which informed firms have an incentive to manipulate an uninformed rival's beliefs over the entry cost by delaying its entry. An informed firm would delay its entry to a date at which, in equilibrium, a sufficient mass of uninformed firms enter if that causes the opponent to believe entry was by an uninformed firm with a sufficiently high likelihood. That way, it would not pay for an uninformed rival to follow suit before having learned the entry cost, which increases the informed entrant's expected profit. We find that the necessary ingredient in such an equilibrium, entry by firms before they have learned the entry cost, is part of an equilibrium strategy if it occurs early in the game, firms are sufficiently impatient, and learning is not too fast. We also find that it is part of an equilibrium strategy for values of the expected entry cost for which it would be optimal for the two firms to experiment until they learn that the entry cost is low before entering the market. Hence, in these equilibria, firms enter excessively early. We furthermore find equilibria in which uninformed firms invest

immediately in order to preempt entry by the rival firm and a waiting equilibrium in which only informed firms enter. The preemption equilibria exist if and only if firms are sufficiently impatient, and learning is not too fast. Because of the common value nature of the entry cost together with the non-profitability of uninformed entry into a duopoly a belief that only low-cost firms enter is self-enforcing. Hence, the equilibrium in which only informed firms enter always exists.

We identify four types of inefficiency from the firms' point of view associated with non-cooperative entry. First, there is excess momentum: firms do not experiment sufficiently to learn the entry cost before investing. Second and third, in every equilibrium, there is entry cost duplication and rent dissipation from competition. Finally, in the waiting equilibrium, there is excess delay. Compared to the private entry cost model set up in Bloch, Fabrizi, and Lippert (2014), there is less excess momentum with common entry costs. First, there is always a waiting equilibrium, which does not always exist for the private values case. Second, the preemption equilibrium exists for a smaller parameter range than in the private values case.

Our analysis relates to the literature on patent races in continuous time, see for example Reinganum (1982) and Harris and Vickers (1985) as well as extensions that allow for symmetric uncertainty due to Spatt and Sterbenz (1985), Harris and Vickers (1987) and Choi (1991). Models of learning in continuous time with public information have been studied by Keller and Rady (1999) and Keller, Rady, and Cripps (2005) in the more complex environment of bandit problems. In contrast, we study private learning. Rosenberg, Solan, and Vieille (2007) and Murto and Välimäki (2011) analyze general stopping games with common values where players' payoffs does not depend on the actions of other players. Similarly, Chamley and Gale (1994), Lambrecht and Perraudin (2003) and Décamps and Mariotti (2004) model market entry with private information about the common value of market entry within a real options framework. We add strategic interaction between the players after entry. Moscarini and Squintani (2010) analyze a common values problem, where agents

learn about the common arrival rate of an innovation in a winner-take-all R&D race. In their model, firms pay a flow investment cost to stay in the race, become more and more pessimistic, and eventually exit. Private information leads to a herding effect where one firm's exit can be followed by the other firm's immediate exit. In our model, firms delay an investment the cost of which they do not know and entry by one firm leads to immediate "me-too" entry of the rival – unless the equilibrium is such that the rival's assigns sufficient probability to uninformed entry at the time of entry.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 derives the cooperative benchmark and Section 4 the non-cooperative equilibria. Section 5 concludes.

2. SETUP

We borrow for our model setup from Bloch et al. (2011) and consider a model with discrete time. We let $t = 1, \ldots, \infty$ denote the periods in the game. The discount factor per period is $\delta = e^{-r\Delta}$ where Δ is the period length and $r > 0$ the pure rate of time preference. At the beginning of the game, nature randomly chooses the *common entry cost* of the two firms, $\theta \in \{\overline{\theta}, \underline{\theta}\}$, but firms do not know the entry cost. For simplicity, assume that nature chooses high and low costs with equal probability.[2] The expected value of the entry cost is thus given by

$$\widetilde{\theta} = \frac{\overline{\theta} + \underline{\theta}}{2}.$$

The information about common entry costs arrives gradually through the game. During the experimentation phase, each firm receives every period a signal $\xi \in \{0, 1, 2\}$. We assume that $\Pr(\xi = 0 | \theta = \underline{\theta}) = \Pr(\xi = 2 | \theta = \overline{\theta}) = \lambda\Delta$, $\Pr(\xi = 1 | \theta = \underline{\theta}) = \Pr(\xi = 1 | \theta = \overline{\theta}) = 1 - \lambda\Delta$ and $\Pr(\xi = 2 | \theta = \underline{\theta}) = \Pr(\xi = 0 | \theta = \overline{\theta}) = 0$ where $\lambda > 0$ is a commonly known parameter, and the period length Δ is small enough so

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[2]The analysis would not change if we assumed different probabilities for the high and low costs.

that $\lambda\Delta < 1$. Hence, with probability $1 - \lambda\Delta$, a firm does not learn the entry cost during the period, and with probability $\lambda\Delta$, a firm receives a perfect signal about the common entry cost. Signals are independent across periods and across players (conditional on the entry cost), and are privately observed by each firm. No payoff is collected by the firms during the experimentation phase.

At each period t, both firms simultaneously make a binary choice, $e_i^t \in \{0, 1\}$. If $e_i^t = 1$, firm i enters the market, pays its entry cost θ, stops the experimentation phase and starts collecting profits. The profits collected by the firm depend on the entry of the other firm. When both firms are present on the market, they each collect a duopoly payoff of $v_d\Delta$ per period. When a single monopolistic firm operates in the market, it collects the monopoly profit $v_m\Delta$ every period. We assume that $v_m > 2v_d$.

We suppose that, with high entry cost, firms never have an incentive to invest, even if they receive monopoly profit. With low entry costs, firms always have an incentive to invest even if they receive duopoly profit. When the entry cost remains unknown, firms have an incentive to invest as monopolists but not as duopolists. Formally, we define the discounted sum of duopoly and monopoly profits as:

$$\pi_d = \frac{\Delta v_d}{1 - e^{-r\Delta}}, \quad \pi_m = \frac{\Delta v_m}{1 - e^{-r\Delta}},$$

and assume:

Assumption 1. $\underline{\theta} \leq \pi_d \leq \widetilde{\theta} \leq \pi_m \leq \overline{\theta}$.

Before analyzing the game played by two competing firms, we compute the profits of leader and follower firms. If the second firm (the follower) only follows suit if it learns that its cost is low, then its expected value is given by

$$V_F = \frac{\delta\lambda\Delta[\pi_d - \underline{\theta}]}{2(1 - \delta(1 - \lambda\Delta))}.$$

If the second firm (the follower) only follows suit if it learns that its cost is low, then gross of the fixed cost, a leader firm that does not know its cost has an expected

payoff of

$$V_L = \pi_m - \frac{\delta\lambda\Delta[\pi_m - \pi_d]}{2(1 - \delta(1 - \lambda\Delta))},$$

whereas a leader firm that knows the entry cost is low has an expected payoff of

$$V_L(\underline{\theta}) = \pi_m - \frac{\delta\lambda\Delta[\pi_m - \pi_d]}{1 - \delta(1 - \lambda\Delta)}.$$

Finally, consider a situation in which no firm has entered and in which the first firm to learn that the entry cost is low invests and is immediately followed by "me-too" entry of the other firm. In this situation, the expected profit of each firm is

$$V_E = \sum_{t=1}^{\infty}(1 - \lambda\Delta)^{2(t-1)}\delta^t\left[(1 - \lambda\Delta)\frac{\lambda\Delta}{2}[\Delta v_m + \delta\pi_d - \underline{\theta}]\right.$$

$$\left. +(1 - \lambda\Delta)\frac{\lambda\Delta}{2}\delta[\pi_d - \underline{\theta}] + \frac{(\lambda\Delta)^2}{2}[\pi_d - \underline{\theta}]\right].$$

Letting the length of every period Δ go to zero, we derive

$$V_F = \frac{\lambda}{2(\lambda + r)}[\pi_d - \underline{\theta}],$$

$$V_L = \pi_m - \frac{\lambda}{2(\lambda + r)}[\pi_m - \pi_d],$$

$$V_L(\underline{\theta}) = \pi_m - \frac{\lambda}{\lambda + r}[\pi_m - \pi_d], \text{ and}$$

$$V_E = \frac{\lambda}{2\lambda + r}[\pi_d - \underline{\theta}].$$

3. Cooperative benchmark

Cooperation requires firms to select one project to enter with. This selection could be a random choice at $t = 0$, leading to a payoff of $\pi_m - \widetilde{\theta}$ or it could be after learning the entry cost from the first project that yields an informative signal. With probability $\frac{1}{2}$, this signal is $\underline{\theta}$, and either it arrives for one project only, either project 1 or project 2, or it arrives for both projects in the same period. Experimenting, therefore, has a payoff of

$$V_{EC} = \sum_{t=1}^{\infty}(1 - \lambda\Delta)^{2(t-1)}\delta^t\left[(1 - \lambda\Delta)\lambda\Delta + \frac{(\lambda\Delta)^2}{2}\right][\pi_m - \underline{\theta}],$$

which, for $\Delta \to 0$ converges to

$$V_{EC} = \frac{\lambda}{2\lambda + r}(\pi_m - \underline{\theta}).$$

Proposition 1. *In the cooperative benchmark, firms experiment if and only if $\pi_m - V_{EC} < \widetilde{\theta}$ and they enter without experimentation otherwise.*

4. ENTRY TIMING

Because entry costs are perfectly correlated, the entry decision of an opponent often carries information about a firm's entry cost and can therefore lead to what we call "me-too" entry of firms that take this decision purely based on the other firm's decision to enter. In this section, we characterize two equilibria in which firms that learned that the cost of entry is low cannot avoid this "me-too" entry.

4.1. **No entry by $\widetilde{\theta}$ firms.** Our first result establishes that there is always a waiting equilibrium where firms only invest after they learn that the cost is low. Because duopoly entry with low entry costs is profitable, immediate "me-too" entry based on the belief that every entry is by firms that learned the entry cost is low, makes a waiting strategy a best reply for each firm.

Proposition 2. *There exists an equilibrium where teams only invest after they learn that the cost is low.*

Proof. In this equilibrium, firms observing entry at any t believe $\underline{\theta}$ and (i) $\widetilde{\theta}$ firms enter as "me-too" entry (with probability 1); (ii) $\widetilde{\theta}$ firms enter with probability 0 otherwise; (iii) $\underline{\theta}$ firms enter immediately after learning their type.

Given the above strategy, a firm that observes entry by the rival assigns probability one to the event that the entry cost is $\underline{\theta}$ so that it is optimal to immediately invest when the rival has done so. The first firm to invest realizes that it will immediately be imitated by the other firm, so that given Assumption 1, the firm will only invest after it has learned that the cost is low. Finally, because the instantaneous payoff

of investing $\pi_d - \underline{\theta}$ is positive, the firm has no incentive to delay investment after it learns that it has a low cost. ∎

The equilibrium in Proposition 2 entails the inefficiencies that have been highlighted by the literature. For $\pi_m - V_{EC} > \widetilde{\theta}$, firms delay their entry excessively, failing to make a timely investment in a profitable market.

4.2. **Entry by $\widetilde{\theta}$ firms without delay by $\underline{\theta}$ firms.**

4.2.1. *One date with entry by $\widetilde{\theta}$ firms without delay by $\underline{\theta}$ firms.* Next, we consider an equilibrium where firms invest with positive probability at some date t before they learn that the cost is low and immediate entry of $\underline{\theta}$ firms. Entry by $\widetilde{\theta}$ firms is only viable if it is not met by "me-too" entry. However, if entry by $\widetilde{\theta}$ firms is not met with "me-too" entry, there will be firms that learn $\underline{\theta}$ sufficiently close to date t that will delay their entry to avoid entry by their rival. The only date at which there is no such delay is $t = 0$. Hence, we state without proof:

Lemma 1. *There is no equilibrium, in which $\widetilde{\theta}$ firms enter with positive probability at one date $t > 0$ and all $\underline{\theta}$ firms enter immediately following their learning of the entry cost.*

Consider an equilibrium where firms invest with positive probability at date $t = 0$ before they learn that the cost is low. In particular, consider the following strategies. (i) Firms that do not know the entry cost ($\widetilde{\theta}$ firms) enter with probability $p_0 \in]0, 1[$ at $t = 0$; (ii) $\widetilde{\theta}$ firms enter immediately following entry by their rival at any $t \neq 0$ ("me-too" entry); (iii) $\widetilde{\theta}$ firms enter with probability 0 otherwise; (iv) $\underline{\theta}$ firms enter immediately after learning their type.

Proposition 3. *An equilibrium, in which $\widetilde{\theta}$ firms enter with positive probability at $t = 0$ and all $\underline{\theta}$ firms enter immediately following their learning of the entry cost exists if and only if $V_L - \widetilde{\theta} \geq V_E$.*

Proposition 3 shows that, for the equilibrium with entry by uninformed firms at $t = 0$ to exist, firms cannot be too patient. The difference $V_L - \widetilde{\theta} - V_E$ is monotonically increasing in r and, if firms were infinitely patient, they would always wait because $\lim_{r \to 0}[V_L - \widetilde{\theta} - V_E] = \frac{\pi_m - \bar{\theta}}{2} < 0$. Proposition 3 also shows that learning must not be too fast. The difference $V_L - \widetilde{\theta} - V_E$ is monotonically decreasing in λ and, for instant learning, firms would always wait because $\lim_{\lambda \to \infty}[V_L - \widetilde{\theta} - V_E] = \frac{\pi_m - \bar{\theta}}{2} < 0$.

In the cooperative benchmark, firms did not experiment as long as $\pi_m - V_{EC} < \widetilde{\theta}$ whereas in the non-cooperative equilibrium established in Proposition 3, firms preempt at $t = 0$ as long as $V_L - V_E < \widetilde{\theta}$. Because $\pi_m - V_{EC} < V_L - V_E$, Proposition 3 implies a non-cooperative solution may entail excess momentum, besides entry cost duplication and rent dissipation. Whether it entails excess momentum depends on the firms' beliefs as there is also the waiting equilibrium established in Proposition 2.

4.2.2. *Continuous entry by uninformed firms without delay by $\underline{\theta}$ firms.* Consider a candidate equilibrium with continuous entry by uninformed firms until some date $\widehat{T}$, immediate entry by $\underline{\theta}$ firms at any time, no me-too entry following entry before $\widehat{T}$ and immediate me-too entry thereafter. We need to check three constraints: $\underline{\theta}$ firms must not have an incentive to delay investment, $\widetilde{\theta}$ firms must not engage in me-too entry before $\widehat{T}$ and do so thereafter, and $\widetilde{\theta}$ firms must be indifferent between entering and not entering before $\widehat{T}$ and prefer to wait until the first firm learns $\underline{\theta}$ thereafter.

Denote by $G(T)$ the cumulative probability that a firm that has not learned its cost has invested by date T. Denote by g the density of G over the interior of its support in the limit as Δ tends to 0.

First, note that $\underline{\theta}$ firms never have an incentive to delay investment. Whether or not they delay entry, $\widetilde{\theta}$ firms do not follow suit before $\widehat{T}$ and follow suit immediately after $\widehat{T}$. Second, we check under which condition a $\widetilde{\theta}$ firm does not immediately follow suit

when they observe entry before $\widehat{T}$. Following suit at time T gives a payoff of

$$\pi_d - \frac{\lambda e^{-\lambda T}}{\lambda e^{-\lambda T} + g(T)}\underline{\theta} - \frac{g(T)}{\lambda e^{-\lambda T} + g(T)}\widetilde{\theta},$$

whereas not following suit gives

$$\frac{\lambda e^{-\lambda T}}{\lambda e^{-\lambda T} + g(T)}\frac{\lambda}{\lambda + r}(\pi_d - \underline{\theta}) + \frac{g(T)}{\lambda e^{-\lambda T} + g(T)}\frac{\lambda}{2(\lambda + r)}(\pi_d - \underline{\theta}).$$

Hence, not following suit is preferred by a $\widetilde{\theta}$ firm if

$$\left(2\lambda e^{-\lambda T} + g(T)\right)\frac{\lambda}{2(\lambda + r)}(\pi_d - \underline{\theta}) > \left(\lambda e^{-\lambda T} + g(T)\right)\pi_d - \lambda e^{-\lambda T}\underline{\theta} - g(T)\widetilde{\theta}$$

or

$$g(T) > \lambda e^{-\lambda T}\frac{(\pi_d - \underline{\theta}) - 2V_F}{V_F - (\pi_d - \widetilde{\theta})}.$$

This can only hold if, at any $T \leq \widehat{T}$, it is sufficiently likely that entry was by an uninformed firm and not by an informed one. This requires that the entry density of informed firms, $\lambda e^{-\lambda T}$, is sufficiently small as compared to the entry density of uninformed firms, $g(T)$, or λ is sufficiently small, implying that learning is sufficiently slow.

Next, we check under which condition an uninformed firm is indifferent between entering and not entering before some date $\widehat{T}$. For that, denote by $\gamma_T(\theta)$ the belief held by a firm about the cost of its rival, conditional on the event that the rival has not invested. Strategies require firms that learn $\underline{\theta}$ to invest immediately, $\gamma_T(\underline{\theta}) = 0$. Using our definition of $G(T)$, we compute beliefs

$$\gamma_T(\overline{\theta}) = \frac{\sum_{t=1}^{\frac{T}{\Delta}}(1 - \lambda\Delta)^{t-1}\frac{\lambda\Delta}{2}[1 - G(\Delta(t-1))]}{\sum_{t=1}^{\frac{T}{\Delta}}(1 - \lambda\Delta)^{t-1}\frac{\lambda\Delta}{2}[1 - G(\Delta(t-1))] + \sum_{t=1}^{\frac{T}{\Delta}}(1 - \lambda\Delta)^{t}[1 - G(\Delta t)]},$$

$$\gamma_T(\widetilde{\theta}) = \frac{\sum_{t=1}^{\frac{T}{\Delta}}(1 - \lambda\Delta)^{t}[1 - G(\Delta t)]}{\sum_{t=1}^{\frac{T}{\Delta}}(1 - \lambda\Delta)^{t-1}\frac{\lambda\Delta}{2}[1 - G(\Delta(t-1))] + \sum_{t=1}^{\frac{T}{\Delta}}(1 - \lambda\Delta)^{t}[1 - G(\Delta t)]}.$$

As $\Delta \to 0$, for any $T < T'$, if $G(T') < 1$, $G(T) = 0$. Hence, as $\Delta \to 0$, beliefs converge to

$$
\begin{aligned}
\gamma_T(\overline{\theta}) &= \frac{1 - e^{-\lambda T}}{1 + e^{-\lambda T}}, \\
\gamma_T(\widetilde{\theta}) &= \frac{2e^{-\lambda T}}{1 + e^{-\lambda T}}.
\end{aligned}
$$

Letting $\Delta \to 0$, a $\widetilde{\theta}$ firm that enters at $\widehat{T}$ has an expected payoff of

$$
\gamma_{\widehat{T}}(\overline{\theta})(\pi_m - \overline{\theta}) + \gamma_{\widehat{T}}(\widetilde{\theta}) \left(g(\widehat{T})\pi_d + (1 - g(\widehat{T}))V_L - \widetilde{\theta} \right),
$$

whereas a $\widetilde{\theta}$ firm that does not enter has an expected payoff at $\widehat{T}$ of

$$
\gamma_{\widehat{T}}(\widetilde{\theta}) \left(g(\widehat{T})V_F + (1 - g(\widehat{T}))V_E \right).
$$

Indifference between entering and not entering implies for $\widehat{T}$

$$
g(\widehat{T}) = \frac{(1 - \gamma_{\widehat{T}}(\widetilde{\theta}))(\pi_m - \overline{\theta}) + \gamma_{\widehat{T}}(\widetilde{\theta})(V_L - \widetilde{\theta}) - V_E}{\gamma_{\widehat{T}}(\widetilde{\theta})(V_L + V_F - \pi_d - V_E)}.
$$

This can only be positive if r is sufficiently large, that is, if firms are sufficiently impatient. Furthermore, because $\gamma_{\widehat{T}}(\widetilde{\theta})$ is decreasing in $\widehat{T}$, it can be positive for $\widehat{T}$ not too large.

Because $\gamma_T(\overline{\theta})$ is increasing over time and $\pi_m - \overline{\theta} < 0$, for equal $g(T)$, the uninformed firm's value of entering is decreasing over time. Furthermore, the value of not entering in earlier periods entails the possibility to enter with a high entry cost in later periods as an uninformed firm. The probability that this happens is higher the more time there is until $\widehat{T}$, which means the value of not entering is increasing over time, implying a decreasing entry probability.

As $\Delta \to 0$, there is (certain) preemption at $t = 0$ with these strategies. Any $\widehat{T} > 0$, irrespective of how small, will do. This establishes the following Proposition.

Proposition 4. *A necessary condition for an equilibrium with certain entry by un-informed firms at $T = 0$ to exist is $V_L - \widetilde{\theta} > V_E$.*

As before, there is excess momentum, here for sufficiently small $\widehat{T}$, there is entry cost duplication and rent dissipation.

4.3. **Entry by $\widetilde{\theta}$ firms with delay by $\underline{\theta}$ firms.** Because an informed firm's entry decision transmits information to a potential competitor, inducing the competitor to enter before it has learned its entry cost, a firm would have an incentive to avoid such entry if they had the opportunity. In this section, we characterize two such equilibria, in which there are discrete dates at which uninformed firms enter with sufficiently large probability that no uninformed firms that did not enter have an incentive to engage in "me-too" entry when they observe entry at those dates. We show that this provides informed firms with an incentive to wait with their entry until those dates.

4.3.1. *One entry date for $\widetilde{\theta}$ firms.* Consider an equilibrium, in which (i) no firm enters at any dates $t \neq \widetilde{t}$, (ii) $\widetilde{\theta}$ firms enter at $\widetilde{t}$ with a probability strictly between 0 and 1, (iii) $\underline{\theta}$ firms enter at $\widetilde{t}$, (iv) $\widetilde{\theta}$ firms do not engage in "me too" entry at $\widetilde{t} + \Delta$, and (v) $\widetilde{\theta}$ firms engage in "me too" entry at $t + \Delta$ if they observe entry at $t \neq \widetilde{t}$.

Let p be prob that type $\widetilde{\theta}$ enters at $\widetilde{t}$ if no earlier entry has occurred. As before, we first check a follower firm's behavior.

Consider a follower firm observing entry at $t > \widetilde{t}$. Given that no $\widetilde{\theta}$ and $\overline{\theta}$ firms enter without prior entry at times $t > \widetilde{t}$, firms immediately update their beliefs to $\underline{\theta}$ if they see entry at those dates. Given this belief, "me-too" entry gives $\pi_d - \underline{\theta} > V_F$ implying that "me-too" entry is an optimal response to entry at $t > \widetilde{t}$.

Consider a follower firm observing entry at $t < \widetilde{t}$. Strategies prescribe that no entry occurs at these dates, hence we are free to assign beliefs. Assigning a belief of 1 to the rival having learned $\underline{\theta}$, "me-too" entry gives $\pi_d - \underline{\theta} > V_F$, which implies that "me-too" entry is an optimal response to entry at $t < \widetilde{t}$.

Next, consider a follower firm that observed entry at date $\widetilde{t} > 0$. At that date, all firms that learned that the entry cost is low before that date enter with probability 1 and all firms that have not yet learned the entry cost enter with probability p.

Therefore, if a firm that does not know the entry cost enters immediately after observing entry at $\widetilde{t}$, it receives

$$\pi_d - \frac{(1 - e^{-\lambda\widetilde{t}})}{(1 - e^{-\lambda\widetilde{t}}) + e^{-\lambda\widetilde{t}}p}\underline{\theta} - \frac{e^{-\lambda\widetilde{t}}p}{(1 - e^{-\lambda\widetilde{t}}) + e^{-\lambda\widetilde{t}}p}\widetilde{\theta}.$$

If it does not enter and instead waits to learn the entry cost, it will eventually learn $\underline{\theta}$ if entry was by a $\underline{\theta}$ firm. If entry was by a $\widetilde{\theta}$ firm, it will learn either $\underline{\theta}$ or $\overline{\theta}$ with probability $\frac{1}{2}$ each, and enter if it learns $\underline{\theta}$. Hence, not entering, it receives

$$\frac{(1 - e^{-\lambda\widetilde{t}})}{(1 - e^{-\lambda\widetilde{t}}) + e^{-\lambda\widetilde{t}}p}\frac{\lambda}{(\lambda + r)}[\pi_d - \underline{\theta}] + \frac{e^{-\lambda\widetilde{t}}p}{(1 - e^{-\lambda\widetilde{t}}) + e^{-\lambda\widetilde{t}}p}\frac{\lambda}{2(\lambda + r)}[\pi_d - \underline{\theta}].$$

A follower firm does not enter if

$$(1) \quad \left[2(1 - e^{-\lambda\widetilde{t}}) + e^{-\lambda\widetilde{t}}p\right]V_F \geq \left[(1 - e^{-\lambda\widetilde{t}}) + e^{-\lambda\widetilde{t}}p\right]\pi_d - (1 - e^{-\lambda\widetilde{t}})\underline{\theta} - e^{-\lambda\widetilde{t}}p\widetilde{\theta}$$

Condition (1) cannot hold for any λ and r if $\widetilde{t}$ is too large: For increasing delay, more and more weight is put on the event that a competitor who learned that there is a low cost of entry, $\underline{\theta}$, has entered such that the value of entering after observing entry at $\widetilde{t}$ is larger than waiting and getting V_F. Indeed, for $\widetilde{t} \to \infty$, "me-too" entry gives $\pi_d - \underline{\theta}$ whereas delaying gives $\frac{\lambda}{\lambda + r}(\pi_d - \underline{\theta})$. Condition (1) holds for any λ and r for $\widetilde{t} \to 0$: For very small delay, it is better not to enter as the probability that a competitor has learned that the cost of entry is low is very small and entry not knowing the cost of entry is not profitable. Indeed, in the extreme of $\widetilde{t} \to 0$, "me-too" entry gives $\pi_d - \widetilde{\theta} < 0$ whereas delay gives $V_F = \frac{\lambda}{2(\lambda + r)}(\pi_d - \underline{\theta}) > 0$.

Next, consider the behavior of a leading firm that has not learned the entry cost (a $\widetilde{\theta}$ firm). Leader entry at dates $t \neq \widetilde{t}$ is followed by immediate "me-too" entry and therefore is unprofitable for $\widetilde{\theta}$ firms: it gives $\pi_d - \widetilde{\theta} < 0$.

To deter "me-too" entry following entry at $\widetilde{t}$, there must be $\widetilde{\theta}$ firm entry at $\widetilde{t}$ with a probability strictly larger than 0 and strictly smaller than 1. If it was 1, then no

firm that learned $\underline{\theta}$ before $\widetilde{t}$ has an incentive to wait until $\widetilde{t}$, and if it was zero, then no firm that learns $\underline{\theta}$ before $\widetilde{t}$ has an incentive to wait until $\widetilde{t}$ because $\widetilde{\theta}$ firms would infer that entry at $\widetilde{t}$ must be by $\underline{\theta}$ firms and therefore would rationally engage in immediate "me too" entry. Hence, $\widetilde{\theta}$ firms must be indifferent between entering and not entering at $\widetilde{t}$.

If a $\widetilde{\theta}$ firm enters at $\widetilde{t}$, the other firm either has learned $\overline{\theta}$ and will never enter, or it has learned $\underline{\theta}$ and enters at $\widetilde{t}$ with probability 1, or it has not learned the entry cost yet and enters with probability p. If it has not learned the entry cost and does not enter, it continues learning and enters if and when it has learned that the entry cost is $\underline{\theta}$. Hence, entering gives

$$\frac{1}{2}(1 - e^{-\lambda\widetilde{t}})[\pi_m - \overline{\theta}] + e^{-\lambda\widetilde{t}}[p\pi_d + (1 - p)V_L - \widetilde{\theta}] + \frac{1}{2}(1 - e^{-\lambda\widetilde{t}})[\pi_d - \underline{\theta}].$$

If the firm does not enter, either the other firm will have entered knowing $\underline{\theta}$ or it will have entered knowing $\widetilde{\theta}$ or it will not have entered knowing $\overline{\theta}$ or it will not have entered knowing $\widetilde{\theta}$. Hence, not entering gives

$$\frac{1}{2}(1 - e^{-\lambda\widetilde{t}})\frac{\lambda}{\lambda + r}[\pi_d - \underline{\theta}] + e^{-\lambda\widetilde{t}}p\frac{\lambda}{2(\lambda + r)}[\pi_d - \underline{\theta}] + e^{-\lambda\widetilde{t}}(1 - p)\frac{\lambda}{2\lambda + r}[\pi_d - \underline{\theta}]$$

or

$$((1 - e^{-\lambda\widetilde{t}}) + e^{-\lambda\widetilde{t}}p)V_F + e^{-\lambda\widetilde{t}}(1 - p)V_E.$$

Indifference of the $\widetilde{\theta}$ firm implies a probability of entry of

$$(2) \qquad p = \frac{(1 - e^{-\lambda\widetilde{t}})\left[\frac{\pi_m + \pi_d}{2} - V_F\right] + e^{-\lambda\widetilde{t}}[V_L - V_E] - \widetilde{\theta}}{e^{-\lambda\widetilde{t}}[V_L + V_F - \pi_d - V_E]}.$$

First, note that for small r, expression (2) is negative. Hence, very patient firms will not enter at $\widetilde{t}$.

Next, note that the numerator of expression (2) is bounded above for any $\widetilde{t}$ whereas the denominator of expression (2) is strictly decreasing in $\widetilde{t}$ with $\lim_{\widetilde{t} \to \infty} e^{-\lambda\widetilde{t}}[V_L + V_F - \pi_d - V_E] = 0$. For large $\widetilde{t}$, entering at $\widetilde{t}$ implies that the opponent is very likely to have received either good or bad news. In case of good news, entry is shared in duopoly. In case of bad news, despite leading to monopoly is unprofitable. A $\widetilde{\theta}$ firm

that does not enter at a large $\widetilde{t}$ will never enter when the cost is high and, thus, avoids unprofitable entry. Hence, entry by uninformed firms must occur sufficiently early.

Finally, note that both denominator and the numerator of expression (2) are increasing in r, that the numerator is increasing in λ whereas the denominator of expression (2) is decreasing in λ, and that $\frac{\pi_m + \pi_d}{2} - V_F - \widetilde{\theta} > 0$ for all λ and r and $V_L - V_E - \widetilde{\theta} > 0$ for suitable combinations of small λ and sufficiently large r. Therefore, for sufficiently small $\widetilde{t}$, we can find λ and r such that $p \in]0, 1[$.

Together, this implies that $p \in]0, 1[$ as long as (i) $\widetilde{t}$ is sufficiently small, (ii) r is sufficiently large, and (iii) λ is sufficiently small.

Lemma 2. *Leading $\widetilde{\theta}$ firms enter at $\widetilde{t}$ if they are sufficiently impatient, learning is slow and firms did not have too much time for learning.*

Now, consider the behavior of a leading $\underline{\theta}$ firm. A firm that learns $\underline{\theta}$ at $t \geq \widetilde{t}$ enters immediately: because no entry by leader $\widetilde{\theta}$ firms occurs after $t = \widetilde{t}$, a rival cannot be manipulated into expecting an entry cost greater than $\underline{\theta}$ by delaying entry. This implies that entry at any date $t > \widetilde{t}$ is met with immediate "me-too" entry by the rival and, hence, waiting is not profitable as it gives the same payoff only delayed.

Finally, consider a firm that learns $\underline{\theta}$ before $\widetilde{t}$. Strategies prescribe this firm to wait until $\widetilde{t}$ with its entry. Assume it learns at $t = \tau$. Then, for $\Delta \to 0$, investing at $t = \tau$ gives $\pi_d - \underline{\theta}$. Waiting until $\widetilde{t}$ delays the payoff. At $\widetilde{t}$, all firms that learned $\underline{\theta}$ until $\widetilde{t}$ enter, whereas $\widetilde{\theta}$ firms enter with probability p and wait to learn the entry cost with probability $1 - p$. Denote by $V_L(\underline{\theta}) = \pi_m - \frac{\lambda}{\lambda + r}[\pi_m - \pi_d]$ the expected payoff of a $\underline{\theta}$ firm that enters at $\widetilde{t}$ if the other firm is a $\widetilde{\theta}$ firm that does not enter at $\widetilde{t}$. Then, waiting gives

$$e^{-r(\widetilde{t} - \tau)} \left((1 - e^{-\lambda \widetilde{t}})\pi_d + e^{-\lambda \widetilde{t}}(p\pi_d + (1 - p)V_L(\underline{\theta})) - \underline{\theta} \right).$$

For a short longest delay, waiting is better than immediate investment: if $\tau \to 0$ and $\widetilde{t} \to 0$, waiting is better than immediate investment if $\pi_d \leq p\pi_d + (1-p)V_L(\underline{\theta})$, which holds for all λ and r. A later $\widetilde{t}$ makes waiting unattractive for two reasons: First, it delays obtaining the market payoff. Second, it increases the likelihood of informed entry by the rival at $\widetilde{t}$, leading to duopoly profits, as does immediate entry. Therefore, for long delay, immediate investment is better than waiting.

Proposition 5. *For sufficiently impatient firms and sufficiently slow learning, there exist equilibria in which $\widetilde{\theta}$ firms invest with positive probability at small $\widetilde{t} > 0$, firms that learned $\underline{\theta}$ before $\widetilde{t}$ invest at $\widetilde{t}$ to conceal their knowledge, and $\widetilde{\theta}$ firms that have not entered at $\widetilde{t}$ do not engage in "me-too" entry.*

4.3.2. *Two entry dates for $\widetilde{\theta}$ firms.* In this subsection, we consider an equilibrium, in which (i) no $\widetilde{\theta}$ firm enters at any dates $t \notin \{\widetilde{t}_1, \widetilde{t}_2\}$, (ii) $\widetilde{\theta}$ firms enter at $\widetilde{t}_1$ with probability $p_1 \in]0, 1[$ and $\widetilde{\theta}$ firms that haven't entered at $\widetilde{t}_1$ enter at $\widetilde{t}_2$ with probability $p_2 \in]0, 1[$, (iii) firms that learned $\underline{\theta}$ before $\widetilde{t}_1$ enter at $\widetilde{t}_1$, firms that learn $\underline{\theta}$ between $\widetilde{t}_1$ and $\widetilde{t}_2$ enter at $\widetilde{t}_2$, and firms that learn $\underline{\theta}$ after $\widetilde{t}_2$ enter immediately, (iv) $\widetilde{\theta}$ firms do not engage in "me too" entry if they observe entry at either $\widetilde{t}_1$ and $\widetilde{t}_2$, and (v) $\widetilde{\theta}$ firms engage in "me too" entry if they observe entry at $t \notin \{\widetilde{t}_1, \widetilde{t}_2\}$. We find that the main constraints for the existence of pre-emption equilibria, that entry by uninformed firms must be sufficiently early, that firms must be sufficiently impatient and that learning must be sufficiently slow, hold also in this equilibrium. All expressions are derived in the Appendix.

As in the equilibrium with one entry date, $\widetilde{t}$, there are three relevant constraints: $\widetilde{\theta}$ firms must not engage in "me-too" entry after observing entry at $\widetilde{t}_1$ and $\widetilde{t}_2$, $\widetilde{\theta}$ firms must be indifferent between entry and no entry at $\widetilde{t}_1$ and $\widetilde{t}_2$, and $\underline{\theta}$ firms must wait until $\widetilde{t}_1$ and $\widetilde{t}_2$, respectively, with their entry.

We find a follower $\widetilde{\theta}$ firm does not enter following entry at $\widetilde{t}_2$ if

$$(3) \quad \left[2(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda\widetilde{t}_2}p_2\right]V_F \geq$$
$$\left[(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda\widetilde{t}_2}p_2\right]\pi_d - (1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})\underline{\theta} - e^{-\lambda\widetilde{t}_2}p_2\widetilde{\theta}$$

and a follower $\widetilde{\theta}$ firm does not enter following entry at $\widetilde{t}_1$ if

$$(4) \quad \left[2(1 - e^{-\lambda\widetilde{t}_1}) + e^{-\lambda\widetilde{t}_1}p_1\right]V_F \geq \left[(1 - e^{-\lambda\widetilde{t}_1}) + e^{-\lambda\widetilde{t}_1}p_1\right]\pi_d - (1 - e^{-\lambda\widetilde{t}_1})\underline{\theta} - e^{-\lambda\widetilde{t}_1}p_1\widetilde{\theta}.$$

Condition (3) cannot hold for any λ and r if $\widetilde{t}_2$ is too large: For increasing delay, more and more weight is put on the event that a competitor who learned that there is a low cost of entry, $\underline{\theta}$, has entered such that the value of entering after observing entry at $\widetilde{t}_2$ is larger than waiting and getting V_F. Indeed, for $\widetilde{t}_2 \to \infty$, "me-too" entry once more gives $\pi_d - \underline{\theta}$, whereas waiting gives $\frac{\lambda}{\lambda+r}[\pi_d - \underline{\theta}]$. Condition (3) holds for any λ and r for $\widetilde{t}_2 \to \widetilde{t}_1$: For very small delay, it is better not to enter as the probability that a competitor has learned that the cost of entry is low is very small and entry not knowing the cost of entry is not profitable. In the extreme of $\widetilde{t}_2 \to \widetilde{t}_1$, "me-too" entry gives $\pi_d - \widetilde{\theta} < 0$ whereas delay gives $V_F = \frac{\lambda}{2(\lambda+r)}[\pi_d - \underline{\theta}] > 0$. For the same reason, condition (4) cannot hold for any λ and r if $\widetilde{t}_1$ is too large and it holds for any λ and r if $\widetilde{t}_1$ is small.

Analogous to the equilibrium with one entry date at which $\widetilde{\theta}$ firms enter, $\widetilde{\theta}$ firms enter at dates $\widetilde{t}_1$ and $\widetilde{t}_2$ with $p_1 \in]0, 1[$ and $p_2 \in]0, 1[$, that is, they have to be indifferent between entering and not entering at both $\widetilde{t}_1$ and $\widetilde{t}_2$. We find indifference between entry and no entry at $\widetilde{t}_2$ implies a probability of entry of

$$(5) \quad p_2 = \frac{\frac{1}{2}(1 - e^{-\lambda\widetilde{t}_1})[\pi_m - \overline{\theta}]}{(1 - p_1)e^{-\lambda\widetilde{t}_2}[V_L + V_F - \pi_d - V_E]}$$
$$+ \frac{(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})\left[\frac{\pi_m + \pi_d}{2} - V_F\right] + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}[V_L - V_E] - \widetilde{\theta}}{e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}[V_L + V_F - \pi_d - V_E]}.$$

We see that we need short delays here (small $\widetilde{t}_1$ and $\widetilde{t}_2$) and sufficiently impatient firms (high r). For a very long first delay, $\widetilde{t}_1$ large, p_2 cannot be larger than 0.

Second, for small $\widetilde{t}_1$ (take the extreme of $\widetilde{t}_1 \to 0$) the expression approaches the one of the model with only one entry date, implying the same conditions for r and $\widetilde{t}_2$ as in that model.

Indifference between entry and no entry at date $\widetilde{t}_1$ implies

$$(6) \qquad p_1 = \frac{(1 - e^{-\lambda \widetilde{t}_1}) \left[\frac{\pi_m + \pi_d}{2} - V_F \right] + e^{-\lambda \widetilde{t}_1}[V_L - **] - \widetilde{\theta}}{e^{-\lambda \widetilde{t}_1}[V_L + V_F - \pi_d - **]},$$

where

$$** := e^{-r(\widetilde{t}_2 - \widetilde{t}_1)}$$

$$\times \left\{ \frac{1}{2}(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) \left[\left((1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}p_2 \right) \pi_d + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}(1 - p_2)V_L(\underline{\theta}) - \underline{\theta} \right] \right.$$

$$\left. + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)} \left[(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})V_F + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}p_2 V_F + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}(1 - p_2)V_E \right] \right\}.$$

For $t_2 \to t_1$, we get

$$** = p_2 V_F + (1 - p_2)V_E$$

and

$$(7) \qquad p_1 = \frac{(1 - e^{-\lambda \widetilde{t}_1}) \left[\frac{\pi_m + \pi_d}{2} - V_F \right] + e^{-\lambda \widetilde{t}_1}[V_L - (p_2 V_F + (1 - p_2)V_E)] - \widetilde{\theta}}{e^{-\lambda \widetilde{t}_1}[V_L + V_F - \pi_d - (p_2 V_F + (1 - p_2)V_E)]}.$$

Again, for small delay $(\widetilde{t}_2 \to \widetilde{t}_1)$, sufficiently slow learning (not too large λ), and sufficiently impatient firms (small r), we have $p_1 \in]0, 1[$.

Note that, for $\widetilde{t}_2 \to \widetilde{t}_1$, any $p_2 > 0$ deters "me-too" entry at $\widetilde{t}_2$, and for $\widetilde{t}_1 \to 0$, any $p_1 > 0$ deters "me-too" entry at $\widetilde{t}_1$. Hence, for $\widetilde{t}_2 \to \widetilde{t}_1$ and $\widetilde{t}_1 \to 0$, p_1 and p_2 satisfy conditions (3) and (4), that is, there is sufficiently likely entry to deter "me-too" entry after $\widetilde{t}_1$ and $\widetilde{t}_2$.

Finally, we find a firm that learns $\underline{\theta}$ at τ with $\widetilde{t}_1 < \tau \le \widetilde{t}_2$ enters at $\widetilde{t}_2$ if

$$\pi_d - \underline{\theta} < e^{-r(\widetilde{t}_2 - \tau)} \left((1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})\pi_d + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}(p_2 \pi_d + (1 - p_2)V_L(\underline{\theta})) - \underline{\theta} \right),$$

and a firm that learns $\underline{\theta}$ at $\tau \le \widetilde{t}_1$ enters at $\widetilde{t}_1$ if

$$\pi_d - \underline{\theta} < e^{-r(\widetilde{t}_1 - \tau)} \left((1 - e^{-\lambda \widetilde{t}_1})\pi_d + e^{-\lambda \widetilde{t}_1}(p_1 \pi_d + (1 - p_1)V_L(\underline{\theta})) - \underline{\theta} \right).$$

In both cases, for a short longest delay, waiting is better than immediate entry. For firms that learn $\underline{\theta}$ between $\widetilde{t}_1$ and $\widetilde{t}_2$, if $\tau \to \widetilde{t}_1$ and $\widetilde{t}_2 \to \widetilde{t}_1$, waiting is better than immediate entry if $\pi_d \leq p_2\pi_d + (1-p_2)V_L(\widetilde{t}_1)$, which holds for all λ and r. As before, a later $\widetilde{t}_2$ makes waiting unattractive for two reasons: First, it delays obtaining the market payoff. Second, it increases the likelihood of informed entry by the rival at $\widetilde{t}_2$, leading to duopoly profits, as does immediate entry. Therefore, for long delay, immediate investment is better than waiting. For firms that learn $\underline{\theta}$ before $\widetilde{t}_1$, for $\tau \to 0$ and $\widetilde{t}_1 \to 0$, waiting is better if $\pi_d \leq p_1\pi_d + (1-p_1)V_L$, which holds for all λ and r. For long delay, waiting is never better than immediate investment: if $\widetilde{t}_1 \to \infty$, it would be better only if $\pi_d \leq 0$, which does not hold for any λ and r.

Note again that entry by uninformed firms is an equilibrium only if $V_L - V_E < \widetilde{\theta}$. As in the other pre-emption equilibria we have characterized, firms need to be sufficiently impatient, learning has to be sufficiently slow, and entry dates for uninformed firms have to be not too distant.

5. Conclusion

This paper studies strategic market entry with learning. Two firms envisage entry into a new market and have the opportunity to gradually learn their common market entry costs. We show that the threat of competition leads firms to attempt to thwart information transmission either by not acquiring information themselves or by delaying their entry until a date at which also uninformed firms enter. Hence, by introducing potential competition, the strategic delay that has been found in previous studies is counteracted by the urge to avoid competition, which leads to both preemptive pressure and delay after learning. As a consequence, we find both excess momentum and excess delay as compared to the optimal cooperative outcome, besides entry cost duplication and the dissipation of monopoly rents.

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APPENDIX A. PROOF OF PROPOSITION 3

Proof. We start with the behavior of a following firm. At $t = 0$, no firm can have learned the entry cost, implying the expected cost of entry is $\widetilde{\theta}$. Letting $\Delta \to 0$, immediate "me-too" entry gives $\pi_d - \widetilde{\theta} < V_F$. Hence "me-too" entry cannot be an optimal response to entry at $t = 0$. Given that no $\widetilde{\theta}$ and $\overline{\theta}$ firms enter without prior entry at times $t \neq 0$, firms immediately update their beliefs to $\underline{\theta}$ if they see entry at those dates. Given this belief, "me-too" entry gives $\pi_d - \underline{\theta} > V_F$ implying that "me-too" entry is an optimal response to entry at $t \neq 0$.

Next, consider the behavior of a leading team. Given our strategies, entry at $t \neq 0$ is followed by immediate "me-too" entry. Letting $\Delta \to 0$, entry at $t \neq 0$ therefore gives $\pi_d - \widetilde{\theta} < 0$ and, thus, $\widetilde{\theta}$ firms do not enter at $t \neq 0$ if their rival has not entered. Consider the condition that $\underline{\theta}$ firms enter immediately. Letting $\Delta \to 0$, immediate

entry at $t = 0$ if a firm learns $\underline{\theta}$ at $t = 0$ (an event with probability approaching zero for $\Delta \to 0$) gives $\pi_m - \frac{\lambda}{\lambda+r}(\pi_m - \pi_d) - \underline{\theta}$ while waiting and entering at $0 + \Delta$ gives $\pi_d - \underline{\theta}$. Clearly, waiting is not profitable. Again letting $\Delta \to 0$, immediate entry at $t \neq 0$ gives $\pi_d - \underline{\theta}$. Given no entry by leader $\widetilde{\theta}$ firms occurs after $t = 0$, the rival cannot be manipulated into expecting a cost of entry greater than $\underline{\theta}$, which implies immediate "me-too" entry. Hence, waiting gives the same payoff only delayed and hence, is not profitable. Finally, consider the condition that $\widetilde{\theta}$ firms enter at $t = 0$ with positive probability. Let p_0 be the probability of entry by $\widetilde{\theta}$ firms at $t = 0$. Letting $\Delta \to 0$, entry at $t = 0$ gives

$$p_0\pi_d + (1 - p_0)[\pi_m - \frac{\lambda}{2(\lambda + r)}(\pi_m - \pi_d)] - \widetilde{\theta} = p_0\pi_d + (1 - p_0)V_L - \widetilde{\theta}.$$

Not entering at $t = 0$ gives

$$p_0\frac{\lambda}{2(\lambda + r)}(\pi_d - \underline{\theta}) + (1 - p_0)\frac{\lambda}{2\lambda + r}(\pi_d - \underline{\theta}) = p_0V_F + (1 - p_0)V_E.$$

A firm is indifferent between entering and not entering if

$$p_0V_F + (1 - p_0)V_E = p_0\pi_d + (1 - p_0)V_L - \widetilde{\theta}$$

resulting in

$$p_0 = \frac{V_L - V_E - \widetilde{\theta}}{V_L + V_F - \pi_d - V_E},$$

which is greater than zero if and only if $V_L - \widetilde{\theta} > V_E$. ∎

APPENDIX B. TWO ENTRY DATES

Start again with a follower firm, which, according to our strategies, must always be a $\widetilde{\theta}$ firm. Assume a firm observes entry at $t > \widetilde{t}_2$. In that case, it updates its beliefs to $\underline{\theta}$ and immediate entry gives $\pi_d - \underline{\theta}$, the largest possible payoff for a follower firm, implying that "me-too" entry is an optimal response to entry at $t > \widetilde{t}_2$.

Consider a follower firm observing entry at $t < \widetilde{t}_2$, $t \neq \widetilde{t}_1$. Strategies prescribe no entry, hence we are free to assign beliefs. Assigning a belief of 1 to the rival having learned $\underline{\theta}$, "me-too" entry gives $\pi_d - \underline{\theta}$, the largest possible payoff for a follower firm, implying again that "me-too" entry is an optimal response to entry at $t < \widetilde{t}_2$, $t \neq \widetilde{t}_1$.

Next consider a follower firm that observed entry at $t = \widetilde{t}_2$. At that date, all firms that learned $\underline{\theta}$ between $\widetilde{t}_1$ and $\widetilde{t}_2$ enter with probability 1 and all firms that have not yet learned the entry cost before that date enter with probability p_2. Therefore, a follower firm that enters immediately after observing entry at $\widetilde{t}_2$ receives

$$\pi_d - \frac{(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})}{(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda\widetilde{t}_2}p_2}\underline{\theta} - \frac{e^{-\lambda\widetilde{t}_2}p_2}{(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda\widetilde{t}_2}p_2}\widetilde{\theta}.$$

A follower firm that does not enter after observing entry at $\widetilde{t}_2$ will learn $\underline{\theta}$ if entry was by a $\underline{\theta}$ firm and enter then. If entry was by a $\widetilde{\theta}$ firm, it will learn either $\underline{\theta}$ or $\overline{\theta}$ with probability $\frac{1}{2}$ each, and enter if it learns $\underline{\theta}$. Hence, a follower firm that does not enter immediately after observing entry at $\widetilde{t}_2$ receives

$$\frac{(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})}{(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda \widetilde{t}_2} p_2} \frac{\lambda}{\lambda + r} [\pi_d - \underline{\theta}] + \frac{e^{-\lambda \widetilde{t}_2} p_2}{(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda \widetilde{t}_2} p_2} \frac{\lambda}{2(\lambda + r)} [\pi_d - \underline{\theta}].$$

Hence, a follower firm does not enter following entry at $\widetilde{t}_2$ if

$$(8) \quad \left[2(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda \widetilde{t}_2} p_2 \right] V_F \geq$$

$$\left[(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda \widetilde{t}_2} p_2 \right] \pi_d - (1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) \underline{\theta} - e^{-\lambda \widetilde{t}_2} p_2 \widetilde{\theta}.$$

Condition (8) cannot hold for any λ and r if $\widetilde{t}_2$ is too large.

Next consider a follower firm that observed entry at $t = \widetilde{t}_1$. At that date, all firms that learned $\underline{\theta}$ before $\widetilde{t}_1$ enter with probability 1 and all firms that have not yet learned the entry cost before that date enter with probability p_1. Therefore, a follower firm that enters immediately after observing entry at $\widetilde{t}_2$ receives

$$\pi_d - \frac{(1 - e^{-\lambda \widetilde{t}_1})}{(1 - e^{-\lambda \widetilde{t}_1}) + e^{-\lambda \widetilde{t}_1} p_1} \underline{\theta} - \frac{e^{-\lambda \widetilde{t}_1} p_1}{(1 - e^{-\lambda \widetilde{t}_1}) + e^{-\lambda \widetilde{t}_1} p_1} \widetilde{\theta}.$$

A follower firm that does not enter after observing entry at $\widetilde{t}_1$ will learn $\underline{\theta}$ if entry was by a $\underline{\theta}$ firm and enter then. If entry was by a $\widetilde{\theta}$ firm, it will learn either $\underline{\theta}$ or $\overline{\theta}$ with probability $\frac{1}{2}$ each, and enter if it learns $\underline{\theta}$. Hence, a follower firm that does not enter immediately after observing entry at $\widetilde{t}_1$ receives

$$\frac{(1 - e^{-\lambda \widetilde{t}_1})}{(1 - e^{-\lambda \widetilde{t}_1}) + e^{-\lambda \widetilde{t}_1} p_1} \frac{\lambda}{\lambda + r} [\pi_d - \underline{\theta}] + \frac{e^{-\lambda \widetilde{t}_1} p_1}{(1 - e^{-\lambda \widetilde{t}_1}) + e^{-\lambda \widetilde{t}_1} p_1} \frac{\lambda}{2(\lambda + r)} [\pi_d - \underline{\theta}].$$

Hence, a follower firm does not enter following entry at $\widetilde{t}_1$ if

$$(9) \quad \left[2(1 - e^{-\lambda \widetilde{t}_1}) + e^{-\lambda \widetilde{t}_1} p_1 \right] V_F \geq \left[(1 - e^{-\lambda \widetilde{t}_1}) + e^{-\lambda \widetilde{t}_1} p_1 \right] \pi_d - (1 - e^{-\lambda \widetilde{t}_1}) \underline{\theta} - e^{-\lambda \widetilde{t}_1} p_1 \widetilde{\theta}.$$

Condition (9) cannot hold for any λ and r if $\widetilde{t}_1$ is too large.

Next, consider the behavior of a leading firm that has not learned the entry cost (a $\widetilde{\theta}$ firm). Again, leader entry at $t \notin \{\widetilde{t}_1, \widetilde{t}_2\}$ is followed by immediate "me-too" entry and therefore is unprofitable for $\widetilde{\theta}$ firms: it gives $\pi_d - \widetilde{\theta} < 0$.

Analogous to the equilibrium with one entry date at which $\widetilde{\theta}$ firms enter, $\widetilde{\theta}$ firms have enter at dates $\widetilde{t}_1$ and $\widetilde{t}_2$ with $p_1 \in]0, 1[$ and $p_2 \in]0, 1[$, that is, they have to be indifferent between entering and not entering at both $\widetilde{t}_1$ and $\widetilde{t}_2$. At $\widetilde{t}_2$, the opponent

either has learned $\overline{\theta}$ and will never enter, or it has learned $\underline{\theta}$ and will enter at $\widetilde{t}_2$ with probability 1, or it has not learned the entry cost yet and will enter with probability p_2. If the opponent has not learned the entry cost and does not enter, it will continue learning and enter if and when it has learned that the entry cost is $\underline{\theta}$. Hence, entering at $\widetilde{t}_2$ gives

$$\frac{\frac{1}{2}(1 - e^{-\lambda \widetilde{t}_1}) + (1 - p_1)e^{-\lambda \widetilde{t}_1}\frac{1}{2}(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})}{\frac{1}{2}(1 + e^{-\lambda t_1}) - p_1 e^{-\lambda \widetilde{t}_1}}[\pi_m - \overline{\theta}]$$

$$+ \frac{(1 - p_1)e^{-\lambda \widetilde{t}_1}\frac{1}{2}(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})}{\frac{1}{2}(1 + e^{-\lambda t_1}) - p_1 e^{-\lambda \widetilde{t}_1}}[\pi_d - \underline{\theta}]$$

$$+ \frac{(1 - p_1)e^{-\lambda \widetilde{t}_2}}{\frac{1}{2}(1 + e^{-\lambda t_1}) - p_1 e^{-\lambda \widetilde{t}_1}}[p_2 \pi_d + (1 - p_2)V_L - \widetilde{\theta}]$$

If the firm does not enter, either the other firm will have entered knowing $\underline{\theta}$ or it will have entered knowing $\widetilde{\theta}$ or it will not have entered knowing $\overline{\theta}$ or it will not have entered knowing $\widetilde{\theta}$. Hence, not entering gives

$$\frac{(1 - p_1)e^{-\lambda \widetilde{t}_1}\frac{1}{2}(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})}{\frac{1}{2}(1 + e^{-\lambda t_1}) - p_1 e^{-\lambda \widetilde{t}_1}}\frac{\lambda}{\lambda + r}[\pi_d - \underline{\theta}]$$

$$+ \frac{(1 - p_1)e^{-\lambda \widetilde{t}_2}}{\frac{1}{2}(1 + e^{-\lambda t_1}) - p_1 e^{-\lambda \widetilde{t}_1}}\left(p_2 \frac{\lambda}{2(\lambda + r)} + (1 - p_2)\frac{\lambda}{2\lambda + r}\right)[\pi_d - \underline{\theta}].$$

Indifference of the $\widetilde{\theta}$ firm implies a probability of entry of

$$(10) \quad p_2 = \frac{\frac{1}{2}(1 - e^{-\lambda \widetilde{t}_1})[\pi_m - \overline{\theta}]}{(1 - p_1)e^{-\lambda \widetilde{t}_2}[V_L + V_F - \pi_d - V_E]}$$

$$+ \frac{(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})\left[\frac{\pi_m + \pi_d}{2} - V_F\right] + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}[V_L - V_E] - \widetilde{\theta}}{e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}[V_L + V_F - \pi_d - V_E]}.$$

We see that we need short delays here (small $\widetilde{t}_1$ and $\widetilde{t}_2$) and sufficiently impatient firms (high r). For a very long first delay, $\widetilde{t}_1$ large, p_2 cannot be larger than 0. Second, for small $\widetilde{t}_1$ (take the extreme of $\widetilde{t}_1 \to 0$) the expression approaches the one of the model with only one entry date, implying the same conditions for r and $\widetilde{t}_2$ as in that model.

At $\widetilde{t}_1$, the opponent either has learned $\overline{\theta}$ and will never enter, or it has learned $\underline{\theta}$ and will enter at $\widetilde{t}_1$ with probability 1, or it has not learned the entry cost yet and will enter with probability p_1. If the opponent has not learned the entry cost and

does not enter, it will continue learning and enter if and when it has learned that the entry cost is $\underline{\theta}$. Hence, entering at $\widetilde{t}_1$ gives

$$\frac{1}{2}(1 - e^{-\lambda \widetilde{t}_1})[\pi_m - \overline{\theta}] + e^{-\lambda \widetilde{t}_1}[p_1 \pi_d + (1 - p_1)V_L - \widetilde{\theta}] + \frac{1}{2}(1 - e^{-\lambda \widetilde{t}_1})[\pi_d - \underline{\theta}].$$

Suppose the firm does not enter at $\widetilde{t}_1$. The either the opponent enters knowing $\underline{\theta}$ or it enters knowing $\widetilde{\theta}$ or it does not enter knowing $\overline{\theta}$ or it does not enter knowing $\widetilde{\theta}$. If the opponent enters at $\widetilde{t}_1$, the firm will wait to learn the entry cost. If the opponent does not enter at $\widetilde{t}_1$, then no entry occurs until $\widetilde{t}_2$. At that time, every $\underline{\theta}$ firm will enter with probability 1 and every $\widetilde{\theta}$ firm will enter with probability p_2. A $\widetilde{\theta}$ firm in particular will be indifferent between entering and not entering and therefore we can write their expected payoff at $\widetilde{t}_2$ as the expected payoff of not entering at $\widetilde{t}_2$. Using all of this, the value of not entering in $\widetilde{t}_1$ is

$$\frac{1}{2}(1 - e^{-\lambda \widetilde{t}_1})\frac{\lambda}{\lambda + r}[\pi_d - \underline{\theta}] + e^{-\lambda \widetilde{t}_1}p_1 \frac{\lambda}{2(\lambda + r)}[\pi_d - \underline{\theta}] + e^{-\lambda \widetilde{t}_1}(1 - p_1)e^{-r(\widetilde{t}_2 - \widetilde{t}_1)}$$

$$\times \left\{ \frac{1}{2}(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) \left[\left((1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}p_2\right)\pi_d + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}(1 - p_2)V_L(\underline{\theta}) - \underline{\theta} \right] \right.$$

$$+ e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)} \left[\frac{1}{2}(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})\frac{\lambda}{\lambda + r}[\pi_d - \underline{\theta}] + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}p_2 \frac{\lambda}{2(\lambda + r)}[\pi_d - \underline{\theta}] \right.$$

$$\left. \left. + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}(1 - p_2)\frac{\lambda}{2\lambda + r}[\pi_d - \underline{\theta}] \right] \right\}$$

Let

$$** := e^{-r(\widetilde{t}_2 - \widetilde{t}_1)}$$

$$\times \left\{ \frac{1}{2}(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) \left[\left((1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}p_2\right)\pi_d + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}(1 - p_2)V_L(\underline{\theta}) - \underline{\theta} \right] \right.$$

$$+ e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)} \left[\frac{1}{2}(1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)})\frac{\lambda}{\lambda + r}[\pi_d - \underline{\theta}] + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}p_2 \frac{\lambda}{2(\lambda + r)}[\pi_d - \underline{\theta}] \right.$$

$$\left. \left. + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}(1 - p_2)\frac{\lambda}{2\lambda + r}[\pi_d - \underline{\theta}] \right] \right\}$$

Then the value of not entering in $\widetilde{t}_1$ is

$$\left((1 - e^{-\lambda \widetilde{t}_1}) + e^{-\lambda \widetilde{t}_1}p_1\right)V_F + e^{-\lambda \widetilde{t}_1}(1 - p_1) \times **.$$

Indifference between entering and not entering in $\widetilde{t}_1$ then implies

(11) $$p_1 = \frac{(1 - e^{-\lambda \widetilde{t}_1})\left[\frac{\pi_m + \pi_d}{2} - V_F\right] + e^{-\lambda \widetilde{t}_1}[V_L - **] - \widetilde{\theta}}{e^{-\lambda \widetilde{t}_1}[V_L + V_F - \pi_d - **]}.$$

For $t_2 \to t_1$, we get

$$** = p_2 V_F + (1 - p_2) V_E$$

and

$$(12) \qquad p_1 = \frac{(1 - e^{-\lambda \widetilde{t}_1}) \left[\frac{\pi_m + \pi_d}{2} - V_F \right] + e^{-\lambda \widetilde{t}_1} [V_L - (p_2 V_F + (1 - p_2) V_E)] - \widetilde{\theta}}{e^{-\lambda \widetilde{t}_1} [V_L + V_F - \pi_d - (p_2 V_F + (1 - p_2) V_E)]}.$$

Again, for small delay $(\widetilde{t}_2 \to \widetilde{t}_1)$, sufficiently slow learning (not too large λ), and sufficiently impatient firms (small r), we have $p_1 \in]0, 1[$.

Now consider a firm that learns $\underline{\theta}$ at τ with $\widetilde{t}_1 < \tau \le \widetilde{t}_2$. Strategies require it to enter at $\widetilde{t}_2$. Clearly, a firm that learns at or before $\widetilde{t}_2$ does not have an incentive to delay further than $\widetilde{t}_2$. If a firm learns at τ with $\widetilde{t}_1 < \tau < \widetilde{t}_2$ then, letting $\Delta \to 0$, entering at τ gives $\pi_d - \underline{\theta}$. Analogous to the model with one entry date for $\widetilde{\theta}$ firms, waiting until $\widetilde{t}_2$ gives

$$e^{-r(\widetilde{t}_2 - \tau)} \left((1 - e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)}) \pi_d + e^{-\lambda(\widetilde{t}_2 - \widetilde{t}_1)} (p_2 \pi_d + (1 - p_2) V_L(\underline{\theta})) - \underline{\theta} \right).$$

For a short longest delay, waiting is better than immediate entry: if $\tau \to \widetilde{t}_1$ and $\widetilde{t}_2 \to \widetilde{t}_1$, waiting is better than immediate entry if $\pi_d \le p_2 \pi_d + (1 - p_2) V_L(\widetilde{t}_1)$, which holds for all λ and r.

Finally, consider a firm that learns $\underline{\theta}$ at $\tau \le \widetilde{t}_1$. Strategies require it to enter at $\widetilde{t}_1$. Once more, clearly, a firm that learns at or before $\widetilde{t}_1$ does not have an incentive to delay further than $\widetilde{t}_1$. If a firm learns at $\tau < \widetilde{t}_1$ then, letting $\Delta \to 0$, entering at τ gives $\pi_d - \underline{\theta}$, whereas waiting until $\widetilde{t}_1$ gives

$$e^{-r(\widetilde{t}_1 - \tau)} \left((1 - e^{-\lambda \widetilde{t}_1}) \pi_d + e^{-\lambda \widetilde{t}_1} (p_1 \pi_d + (1 - p_1) V_L(\underline{\theta})) - \underline{\theta} \right).$$

Again, for a short longest delay, waiting is better than immediate entry: Analogous to before, for $\tau \to 0$ and $\widetilde{t}_1 \to 0$, it is better if $\pi_d \le p_1 \pi_d + (1 - p_1) V_L$, which holds for all λ and r. For long delay, waiting is never better than immediate investment: if $\widetilde{t}_1 \to \infty$, it would be better only if $\pi_d \le 0$, which does not hold for any λ and r.