Double-Clock Auctions and Two-sided VCG with Reserves

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Introduction

- Market making – balancing demand and supply by quoting market clearing prices – is at the heart of economics.
- Can a market maker elicit the necessary information from savvy traders?
- Vickrey, Hurwicz, Myerson and Satterthwaite gave negative answers.

Deficit under Efficiency

Vickrey (1961):

- market with homogenous goods, multi-unit traders with substitutes preferences
- Vickrey first developed the efficient dominant-strategy mechanism,
- noting that this scheme results in a deficit and would therefore be **“inordinately expensive”** for the market maker.

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Vickrey (1961) Revisited

It is tempting to try to modify this scheme in various ways that would reduce or eliminate this cost of operation ... However, it seems that all modifications that do diminish the cost of the scheme either imply the ***use of some external information*** *as to the true equilibrium price or reintroduce a direct incentive for* ***misrepresentation*** *... in some cases the impairment of optimum allocation would be small relative to the reduction in cost, but, unfortunately, the analysis of such variations is extremely difficult; ...*

This paper

- We address the challenges identified by Vickrey.
- For the same canonical model, we develop a **double-clock auction (DCA)** that
  - endows agents with dominant strategies
  - never runs a deficit
  - satisfies ex post IR
  - is envy-free
  - and under regularity conditions converges to full efficiency as the market becomes large, using an estimation-based tâtonnement process.

Sketch of DCA

- Two phases:
  1. **discovery phase**: reserve price and aggregate quantity traded are discovered
  2. **allocation phase**: this quantity is allocated using two Ausubel (2004) auctions with reserves, yielding VCG prices bound by the reserves.

- during the discovery phase, whether the buyers' or the sellers' clocks (or both) move depends on the sign of **estimated excess demand**.

- agents indicate their "activity" (quantity demanded or supplied) – which cannot increase – at any given price

Flexible Design

- no common prior required; the system of equilibrium prices contains more information than any individual has, much as argued by Hayek (1945)
- accommodates other objectives, such as revenue, which matters for electronic trading platforms and beyond (e.g. for the FCC's "incentive auction")
- permits quantity constraints (such as caps), which are often relevant (e.g. because of competition concerns).
- clock auctions preserve privacy of agents (insurance against bidders' hold-up by designer)

Related Literature

- DS-tâtonnement in one-sided setups: Ausubel (2004, 2006), Sun and Yang (2009, 2014)
- double-auctions: Chatterjee and Samuelson (1983), Wilson (1985), Gresik and Satterthwaite (1989), Rustichini, Satterthwaite and Williams (1994), Satterthwaite and Williams (1989, 2002), Cripps and Swinkels (2006), Satterhtwaite, Williams and Zachariadis (2014, 2015)
- McAfee (1992); Vickrey (1961), Clarke (1971), Groves (1973); Hagerty and Rogerson (1987); Tatur (2005), Yoon (2001, 2008); Kojima and Yamashita (2014) [ex post equilibrium]
- mechanism design with estimation: Baliga and Vohra (2003), Segal (2003); Loertscher and Marx (2015).

Setup

- homogenous good
- N_B buyers b and N_S sellers s
- $v_k^b \in [0, 100]$ is buyer b's marginal value for the kth unit with $k = 1, .., K_B$
- $c_k^s \in [0, 100]$ is seller s's cost for selling the kth unit with $k = 1, .., K_S$

Setup

- decreasing marginal values $v_k^b \geq v_{k+1}^b$
- increasing marginal costs $c_k^s \leq c_{k+1}^s$
- buyer b's payoff from buying q units at prices $\mathbf{p}^b = (p_1^b, ..., p_q^b)$ is

$$\sum_{k=1}^{q} \left(v_k^b - p_k^b \right)$$

and seller s's payoff from selling q units at prices $\mathbf{p}^s = (p_1^s, ..., p_q^s)$ is

$$\sum_{k=1}^{q} (p_k^s - c_k^s) \, .$$

VCG-Mechanism and VCG-Prices

- **VCG-mechanism**:
  1. collects reports $(\boldsymbol{v}, \boldsymbol{c})$, determines **personalized prices**:

$$\begin{aligned} \mathbf{p}^b(\boldsymbol{\theta}) &:= (p_1^b, .., p_{\overline{Q}_S}^b) = (\theta_{(\overline{Q}_S)}^{-b}, .., \theta_{(1)}^{-b}) \\ \mathbf{p}^s(\boldsymbol{\theta}) &:= (p_1^s, .., p_{\overline{Q}_B}^s) = (\theta_{[\overline{Q}_B]}^{-s}, .., \theta_{[1]}^{-s}). \end{aligned}$$

  2. given $\mathbf{p}^b(\boldsymbol{\theta})$ and $\mathbf{p}^s(\boldsymbol{\theta})$, b and s chooses optimal quantities $q^b \in \{0, .., \overline{Q}_S\}$ and $q^s \in \{0, .., \overline{Q}_B\}$.
- the VCG-mechanism endows agents with dominant strategies (DS) to report truthfully and choose the efficient quantities given reports

Deficit on every unit traded under efficiency

Proposition

(i) *In the DS equilibrium of the VCG-mechanism all the unit prices buyers pay are uniformly smaller than the unit prices sellers are paid whenever* $Q^W(\mathbf{v}, \mathbf{c}) > 0$.

(ii) *Any IR and DS mechanism that always induces the Walrasian allocation runs a deficit whenever* $Q^W(\mathbf{v}, \mathbf{c}) > 0$.

(▸ Go to Proof of (i) and (ii))

Statement of the Problem

- Problem inherent to the two-sided setup - deficits on every trade under efficiency.
- It motivates to use reserves to avoid deficits.
- Reserves need to be endogenous to avoid disasters.
- Nontrivial if one wants to maintain the DS property.
- In contrast to one-sided setups, the quantity to be traded is not known at the outset.

DCA

- increasing buyers' clock p_B starting at 0
- decreasing sellers' clock starting at $p_S = 100$
- agents indicate activity – quantities they demand or supply
- activity is restricted to be weakly decreasing throughout the DCA

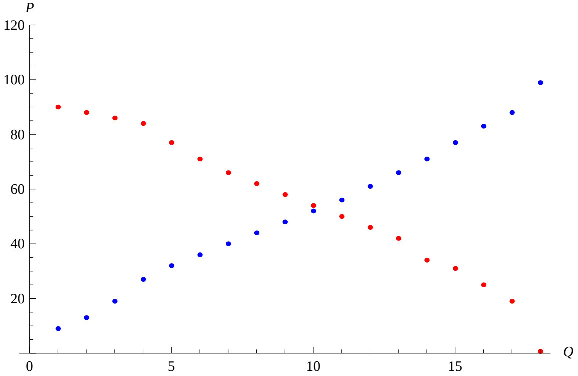
Example: Valuations

Valuations			
Buyer 1	90	86	71
Buyer 2	88	58	37
Buyer 3	*84*	77	25
Buyer 4	66	**54**	46
Buyer 5	62	42	0.7
Buyer 6	50	0.6	0.5
Buyer 7	31	0.4	0.3
Buyer 8	19	0.2	0.1

Example: Costs

Costs						
Seller 1	9	36		Seller 8	48	99.3
Seller 2	13	56		Seller 9	**52**	99.4
Seller 3	19	98.9		Seller 10	66	99.5
Seller 4	*27*	61		Seller 11	71	99.6
Seller 5	32	99		Seller 12	77	99.7
Seller 6	40	99.1		Seller 13	83	99.8
Seller 7	44	99.2		Seller 14	88	99.9

Market demand and supply



Efficient quantity is 10 and the Walrasian price interval is $[52, 54]$.

Discovery Phase: Estimation

- each time a buyer (seller) exits, market demand (supply) is estimated anew (using OLS in the example).
- initial estimates are $D_0^{est}(p) = 24 - 6p/25$ and $S_0^{est}(p) = 7p/25$.
- remaining active agents will be assumed to be active at full activity in the estimation (to fix ideas, not required).
- suppose Buyer 8 is the first to drop out at a price of 19 bidding sincerely.
- regressing the "observed" quantities demanded $\boldsymbol{y} = (24, 23, 23, 22, 22, 21)$ onto a constant and prices $\boldsymbol{p} = (0.1, 0.11, 0.2, 0.21, 19, 19.01)$ (discretizing price changes by 0.01) yields estimated demand $D_1^{est}(p) = 23.01 - 0.079p$.

Discovery Phase: Target Prices

- Assume $n \geq 0$ buyers and $m \geq 0$ sellers have dropped out and that the standing clock prices are p_B and p_S with $p_B < p_S$.
- DCA computes a **target price** p^T satisfying $p^T \geq p_B$ and $p^T \leq p_S$ and
  - (i) $D_n^{est}(p^T) = S_m^{est}(p^T)$ if $D_n^{est}(p_B) - S_m^{est}(p_S) = 0$
  - (ii) $D_n^{est}(p^T) = S_m^{est}(p_S)$ if $D_n^{est}(p_B) - S_m^{est}(p_S) > 0$
  - (iii) $D_n^{est}(p_B) = S_m^{est}(p^T)$ if $D_n^{est}(p_B) - S_m^{est}(p_S) < 0$
- In case (i), both clocks move; in case (ii) only the buyers' clock moves while in case (iii) only the sellers' clock moves.
- Clocks stop moving at the earlier of the two events: the target price is met or another agent becomes inactive.

Discovery Phase

round	$\#_B$	$\#_S$	p_B	p_S	p^T	est.exc.dem.	clock(s)
1	0	0	0	100	85.71	**-4**	S
2	0	**1**	0	**88**	58.28	-2.5	S
3	0	**2**	0	**83**	*78.70*	-0.62	S
4	0	2	0	**78.70**	29.40	**0**	BOTH
5	0	**3**	1.01	**77**	*6.28*	1.27	B
6	0	3	**6.28**	77	36.23	**0**	BOTH
7	0	**4**	10.69	**71**	*15.33*	1.12	B
8	0	4	**15.33**	71	40.38	**0**	BOTH
9	**1**	4	**19**	66.52	44.78	2.06	B
10	**2**	4	**31**	66.52	60.86	-1.11	S
11	2	**5**	31	**66**	*31.69*	0.08	B

From Discovery to Allocation Phase

- Discovery phase ends after finitely many steps when the target price is reached with $p_B \geq p_S$ (or no active traders remain on one side).
- Target price becomes the reserve price.
- Aggregate quantity traded is the minimum of *true* demand and *true* supply at the reserve (*true* as opposed to estimated).
- **Allocation phase**: short side trades at the reserve while the long side participates in an Ausubel auction with this reserve, yielding VCG prices bound by the reserve.

End of Discovery Phase

round	$\#_B$	$\#_S$	p_B	p_S	p^T	est.exc.dem.	clock(s)
12	2	5	**31.69**	66	53.71	**0**	BOTH
13	**3**	5	**50**	55.78	50	-1.36	S
14	3	**6**	50	**52**	*50.10*	-0.38	S
15	3	6	50	**50.10**	*50.06*	**0**	BOTH
16	3	6	**50.06**	**50.06**	50.06	0	END

Allocation Phase

- At $r = 50.06$, $D(r) = 10 > 9 = S(r)$.
- All active sellers thus trade their units at the reserve.
- The equilibrium quantity traded falls one short of the efficient quantity (but per capita efficiency loss converges to 0 in the large).
- Buyer 1 clinches three units at prices $(50.06, 50.06, 54)$.
- Buyers 2 and 3 clinch two units each at prices $(50.06, 54)$.
- Buyer 4 and 5 clinch one unit each, at prices of 50.06 and 54, respectively.

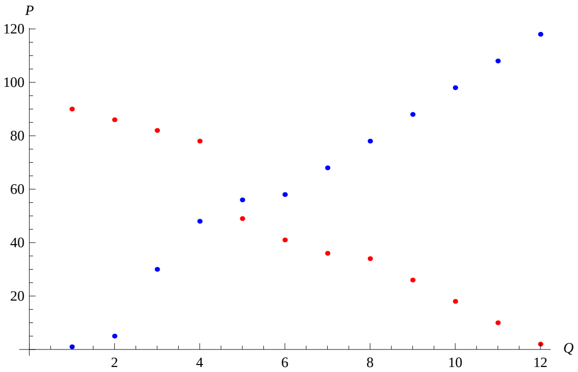
Properties

Buyer b is said to **bid sincerely** if for any price p her demand is $h(p)$ where $h(p)$ is such that $v^b_{h(p)} \geq p > v^b_{h(p)+1}$; similarly for sellers.

Theorem

Sincere bidding is a dominant strategy for each agent in the DCA. The DCA also satisfies ex-post individual rationality and is both deficit free and envy free.

Marginal Revenue and Marginal Procurement Cost



The revenue maximizing quantity is 4, traded at prices $p_B = 84$ and $p_S = 27$.

Revenue Targeting Version of the DCA

- Estimated demand $D_n^{est}(p)$ and supply $S_m^{est}(p)$ imply estimates of marginal revenue $MR_n^{est}(p)$ and marginal procurement costs $MC_m^{est}(p)$.
- The efficiency targeting version of the DCA can then be amended to target revenue.
- Essentially, we replace the target price p^T by a pair of buyers' and sellers' target prices (p_B^T, p_S^T) that equate

$$D_n^{est}(p_B^T) = S_m^{est}(p_S^T) \quad \text{and} \quad MR_n^{est}(p_B^T) = MC_m^{est}(p_S^T)$$

after n buyers and m sellers have dropped out.

Discovery Phase of the Revenue Targeting DCA

round	$\#_B$	$\#_S$	p_B	p_S	p_T^B	p_T^S	exc. dem.	clock(s)
1	0	0	0	100	73.08	23.08	**-4**	S
2	0	**1**	0	**88**	57.56	0	-2.5	S
3	0	**2**	0	**83**	64.70	0	-0.62	S
4	0	**3**	0	**77**	68.11	0	1.51	B
5	**1**	3	**19**	77	100	2.22	-1.00	S
6	1	**4**	19	**71**	100	13.87	1.18	B
7	**2**	4	**31**	71	100	8.56	-2.00	S
8	2	**5**	31	**66**	100	16.95	0.08	B
9	**3**	5	**50**	66	100	14.91	-3.53	S
10	3	**6**	50	**52**	100	13.56	-0.38	S
11	3	**7**	50	**48**	100	17.16	1.19	B

Discovery Phase of the Revenue Targeting DCA

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12	**4**	7	**62**	48	97.71	15.26	-1.75	S
13	4	**8**	62	**44**	99.02	19.84	0.02	B
14	**5**	8	**66**	44	87.51	18.43	-1.94	S
15	5	**9**	66	**40**	88.76	22.97	-0.06	S
16	5	**10**	66	**32**	89.76	26.21	2.55	B
17	**6**	10	**84**	32	84.20	25.27	-1.48	S
18	6	**11**	84	**27**	84	27	0.51	END

From Discovery to Allocation Phase

- As $D(84) = 3 = S(27)$, there is no short side and the **allocation phase** becomes trivial – all active agents trade at their sides' reserve.
- The reserve prices are the same that maximize revenue ex post but the quantity traded is one short of the revenue maximizing quantity.
- In the large, we show that the percentage loss in profits goes to 0.

Quantity constraints

- Quantity constraints limit the number of units some bidders may acquire.
- Often imposed because of competition concerns (within the mechanism or in downstream markets; e.g. AT & T, Verizon).
- Our DCA can accommodate such constraints.
- Essentially, first solve the one-sided allocation problem involving the constrained bidders.

Asymptotics

- DS implies: no information from active agents and all information from dropped out agents can be used.
- Estimation thus has to be based on data from least efficient (w.r.t. first units) agents.
- Need to know/infer: $D(p_B)$ and $S(p_S)$ conditional on history up to p_B and p_S.
- Interestingly, we don't need to extrapolate $D(p_B)$ and $S(p_S)$ to other prices.
- Rather, need ability to gauge $D(p_B)$ and $S(p_S)$ in multi-unit setup based on less efficient guys' behavior.

Asymptotics: Regularity Conditions

Regularity conditions under which we get convergence to optimality (percentage loss goes to zero):

- Monotonicity of demand and supply (for efficiency) and of marginal revenue and marginal procurement cost (for revenue)
- Bounded marginal-to-joint variation ratio (roughly, need not only estimates of $F_{(1)}(p_B)$ and $G_{[1]}(p_S)$ but of $F_{(k)}(p_B)$ and $G_{[k]}(p_S)$ for all k).

Conclusions

- Market making and price discovery using an estimation-based tâtonnement process within a double-clock auction.
- Dominant strategies (or price-taking behaviour) maintained by making the mechanism essentially only a function of behaviour of agents who drop out.
- Future work: multiple goods/markets; differentiated goods.

Proof of (i) and (ii)

(i) suppose b consumes q units under efficiency:

$$p_q^b = \theta^{-b}_{(\overline{Q}_S-(q-1))} \leq \theta_{(\overline{Q}_S+q-(q-1))} = \theta_{(\overline{Q}_S+1))}$$

(with strict inequality if $v_h^b \geq \theta_{(\overline{Q}_S+1)}$ for some $h > q$). similarly, for seller s who optimally procudes q units:

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since

$$\theta_{[\overline{Q}_B+1]} > \theta_{(\overline{Q}_S+1)}$$

because θ has $\overline{Q}_S + \overline{Q}_B$ elements, (i) holds.

(ii) follows by payoff equivalence and because VCG is DS and satisfies IR by giving 0 to agents who do not trade.

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