Equilibrium, uncertainty and risk in hydro-thermal electricity systems[1]

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(joint work with Roger Wets and Michael Ferris)

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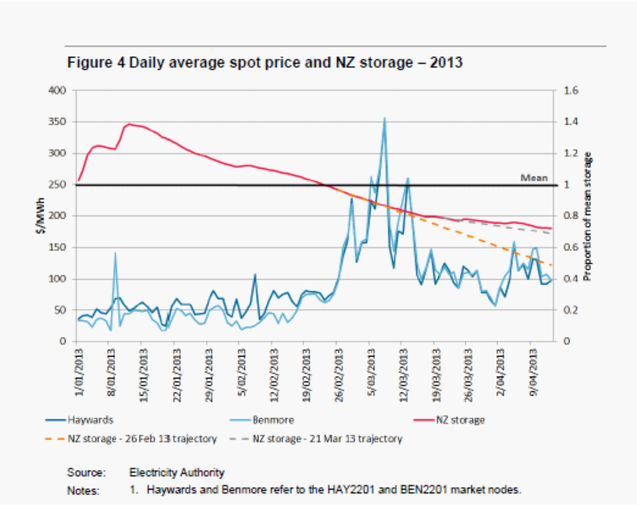
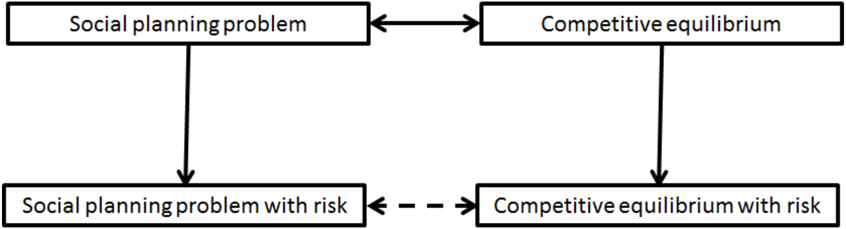
New Zealand Electricity Authority Report, July 29, 2013



Some questions about risk

- Are high electricity prices reflecting shortage risk?
- What does it mean to say that hydro generators are being too risk averse?
- What does shortage risk mean for a single-buyer model?

Our framework for discussion



Classical (dis)utility theory
(Von Neumann, Morgenstern,1947)

Throughout we consider losses (i.e. disbenefit) as primitive. Utility theory models risk using increasing (typically convex) disutility function u. The risk $\rho(Z)$ of a random loss Z with distribution F is modelled as

$$\rho(Z) = \mathbb{E}[u(Z)] = \int u(z)dF(z).$$

Main difficulty

Dynamic programming optimization is difficult with utility theory because in general $\rho(c+Z) \neq c + \rho(Z)$

Rank-dependent (dis)utility
(Quiggin,1982, Yaari, 1987)

Apply some (convex) function to the probability distribution. The risk $\rho(Z)$ of a random disbenefit Z with distribution F is modelled as

$$\rho(Z) = \mathbb{E}_{G(F)}[Z] = \int z dG(F(z)).$$

Now we have translation equivariance:

$$\rho(c + Z) = c + \rho(Z)$$

which makes dynamic programming straightforward if G is known.

Coherent risk measures
(Artzner et al ,1999)

A coherent risk measure is a mapping ρ from a space $\mathbb{Z}$ of random variables to $\mathbb{R}$ that satisfies the following axioms for Z_1 and $Z_2 \in \mathbb{Z}$.

Subadditivity: $\rho(Z_1 + Z_2) \leq \rho(Z_1) + \rho(Z_2)$;

Monotonicity: If $Z_1 \leq Z_2$, then $\rho(Z_1) \leq \rho(Z_2)$;

Positive homogeneity: If $c \in \mathbb{R}$ and $c > 0$, then

$$\rho(cZ_1) = c\rho(Z_1);$$

Translation equivariance: If $c \in \mathbb{R}$, then

$$\rho(c + Z_1) = c + \rho(Z_1).$$

Dual representation

A coherent risk measure of a random disbenefit Z can be expressed as

$$\rho(Z) = \max_{\mu \in \mathcal{D}} \mathbb{E}_{\mu}[Z]$$

where $\mathcal{D}$ is a convex set of probability measures called the risk set.

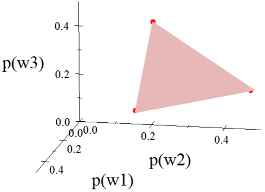
Example: three outcomes

Consider possible cost outcomes

$$Z(\omega_1) < Z(\omega_2) < Z(\omega_3)$$

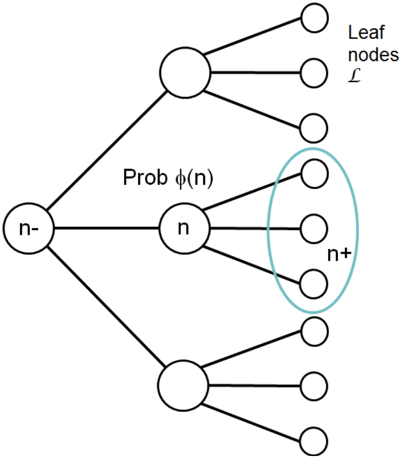
Let

$$\mathcal{D}=\text{conv}\{(\frac{1}{2},\frac{1}{4},\frac{1}{4}),(\frac{1}{4},\frac{1}{2},\frac{1}{4}),(\frac{1}{4},\frac{1}{4},\frac{1}{2})\}$$



$$\rho(Z) = \max_{\mu\in\mathcal{D}} \mathbb{E}_\mu[Z] = \frac{1}{4}Z(\omega_1) + \frac{1}{4}Z(\omega_2) + \frac{1}{2}Z(\omega_3)$$

Multi-stage optimization and risk



Each node n corresponds to a realization $\omega(n)$ of reservoir inflows.

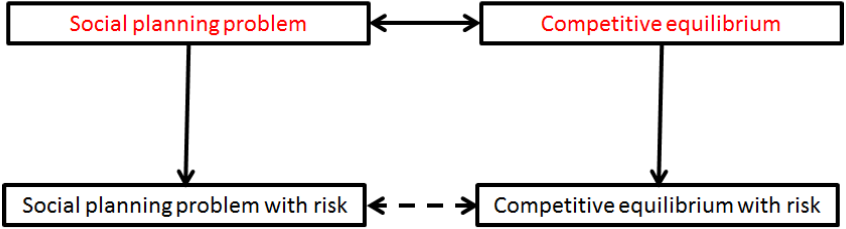
Dynamic risk measures
(Ruszczynski, 2010)

Consider a random sequence of costs $Z(n)$ that is adapted to the filtration defined by the scenario tree. Each node $n \in \mathcal{N} \setminus \mathcal{L}$ in the scenario tree is endowed with a risk set $D(n)$. The dynamic risk measure we will use is constructed recursively as follows. For every leaf node we set the risk-adjusted cost

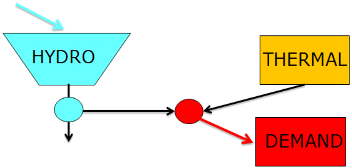
$$\rho(n) = Z(n)$$

and for every other node we set

$$\rho(n) = Z(n) + \max_{\mu \in \mathcal{D}(n)} \sum_{m \in n+} \mu(m)\rho(m).$$


Social planning problem
Competitive equilibrium
Social planning problem with risk
Competitive equilibrium with risk

Social plan minimizes total expected system disbenefit



$$\min \quad \sum_{n\in\mathcal{N}} \phi(n) \left(\sum_{j\in\mathcal{T}} C_j(v_j(n)) - \sum_{c\in\mathcal{C}} D_c(d_c(n))\right) - \sum_{n\in\mathcal{L}} \phi(n) \sum_{i\in\mathcal{H}} V_i(x_i(n))$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{H}} g_i(u_i(n)) + \sum_{j\in\mathcal{T}} v_j(n) \geq \sum_{c\in\mathcal{C}} d_c(n), \qquad n \in \mathcal{N}$$

$$x_i(n) = x_i(n-) - u_i(n) - s_i(n) + \omega_i(n), \qquad i \in \mathcal{H}, n \in \mathcal{N}.$$

Social plan = risk neutral perfectly competitive equilibrium

To minimize Lagrangian for social plan with Lagrange multipliers $\phi(n)p(n)$ we solve each agent problem separately.

$$\begin{aligned} \text{HP}(i)\text{:max} \quad & \sum_{n\in\mathcal{N}} \phi(n)p(n)g_i(u_i(n)) + \sum_{n\in\mathcal{L}} \phi(n)V_i(x_i(n)) \\ \text{s.t.} \quad & x_i(n) = x_i(n-) - u_i(n) - s_i(n) + \omega_i(n), \qquad n \in \mathcal{N}, \\ & u_i(n), x_i(n), s_i(n) \geq 0, \end{aligned}$$

$$\begin{aligned} \text{TP}(j)\text{: max} \quad & \sum_{n\in\mathcal{N}} \phi(n)p(n)(v_j(n) - C_j(v_j(n))) \\ \text{s.t.} \quad & v_j(n) \geq 0, \end{aligned}$$

$$\begin{aligned} \text{CP}(c)\text{: max} \quad & \sum_{n\in\mathcal{N}} \phi(n)\left(D_c(d_c(n)) - p(n)d_c(n)\right) \\ \text{s.t.} \quad & d_c(n) \geq 0. \end{aligned}$$

Social plan = risk neutral perfectly competitive equilibrium

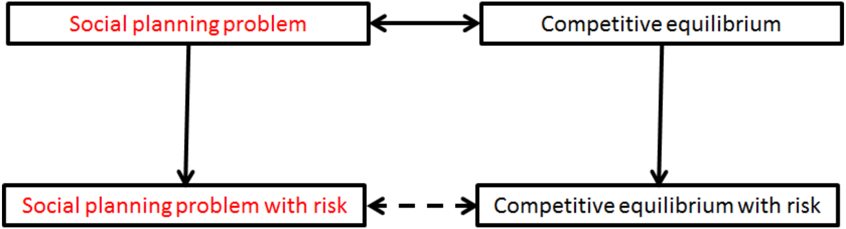
This defines a perfectly competitive equilibrium defined by the individual optimality conditions and market clearing condition.

$$\text{CE:}\quad u_i, x_i, s_i \in \arg\max \mathrm{HP}(i),$$

$$v_j(n) \in \arg\max \mathrm{TP}(j),$$

$$d_c(n) \in \arg\max \mathrm{CP}(c),$$

$$0 \leq \sum_{i \in \mathcal{H}} g_i(u_i(n)) + \sum_{j \in \mathcal{T}} v_j(n) - \sum_{c \in \mathcal{C}} d_c(n) \perp p(n) \geq 0.$$


Social planning problem
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Social planning with a coherent risk measure
(Philpott and de Matos, 2011, 2013, Shapiro 2012)

Risk aversion controlled using a parameter $\lambda \in [0, 1)$ that determines a risk set at each node. We sample M inflow outcomes per stage, and set $a = \frac{1+(M-1)\lambda}{M}$, $b = \frac{1-\lambda}{M}$ Then

$$\mu \in \mathcal{D}(n) = \text{conv}\{(a, b, \ldots, b, b), (b, a, b, \ldots, b, b), \ldots, (b, b, \ldots, b, a)\}$$

For example

- if $M = 3$, and $\lambda = 0.25$ then

$$a = \frac{1}{2}, \quad b = \frac{1}{4}$$

- if $M = 10$, and $\lambda = 0.2$ then

$$a = 0.28, \quad b = 0.08.$$

Approximate problem at node n of scenario tree

(Philpott and de Matos, 2013)

Stage problem for each node $m \in n+$ is approximated by

$$
\begin{aligned}
Q_m(x(n), m) = \min \quad & \left(\sum_{j\in\mathcal{T}} C_j(v_j(n)) - \sum_{c\in\mathcal{C}} D_c(d_c(n))\right) + \theta_m \\
\text{s.t.} \quad & \sum_{i\in\mathcal{H}} g_i(u_i(n)) + \sum_{j\in\mathcal{T}} v_j(n) \geq \sum_{c\in\mathcal{C}} d_c(n), \\
& x_i(m) = x_i(n) - u_i(n) - s_i(n) + \omega_i(m), \\
& \theta_m \geq \alpha_k + \beta_k^\top x(m), \quad k = 1, 2, \ldots, K.
\end{aligned}
$$

Here θ_m is a variable giving an outer approximation of the risk adjusted future cost $\rho(m)$ in child node m.

$$\beta_k = \mathbb{E}_{\mu^*}[\nabla_x Q_m(x^k(n), m))], \quad \alpha_k = \mathbb{E}_{\mu^*}[Q_m(x^k(n), m)] + \beta^\top x^k(m-)$$

where

$$\mu^* \in \arg\max_{\mu\in\mathcal{D}(n)} \sum_{m\in n+} \mu(m) Q_m(x^k(n), m).$$

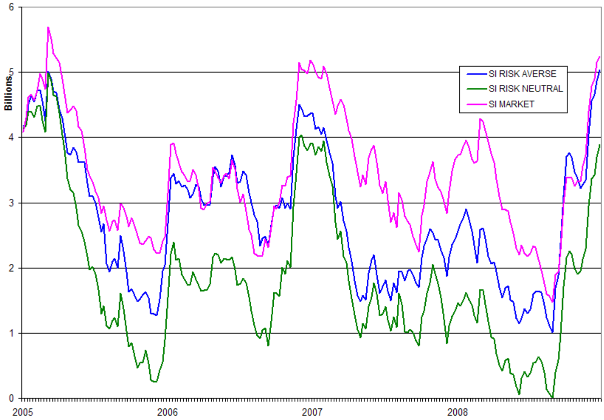
DOASA=Dynamic outer approximation sampling algorithm

(Philpott and Guan, 2008, Based on SDDP, Pereira and Pinto, 1991)

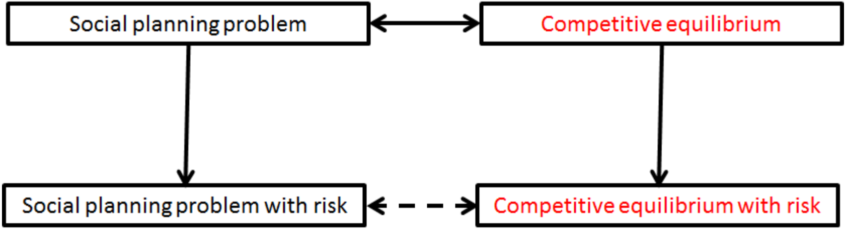
We can apply the above approximation in a SDDP scheme. Our code DOASA finds an approximately optimal solution to the risk-averse social planning problem using dynamic programming. This defines:

- a candidate policy for the social plan defined by a risk-adjusted Bellman function;
- a lower bound on the risk-adjusted value of an optimal policy for the social plan.

Example: New Zealand reservoir storage with risk aversion



Risk averse social plan computed using DOASA and simulated in blue and compared with market trajectory (pink) and the trajectory of a risk-neutral social plan (in green).


Social planning problem
Competitive equilibrium
Social planning problem with risk
Competitive equilibrium with risk

Recall dynamic risk measure

For agent $a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}$ consider a random sequence of costs $Z_a(n)$ defined for each node of the scenario tree. Each agent a at each node $n \in \mathcal{N} \setminus \mathcal{L}$ in the scenario tree is endowed with a risk set $\mathcal{D}_a(n)$. The dynamic risk measure we will use for a is constructed recursively as follows. For every leaf node we set

$$\rho_a(n) = Z_a(n)$$

and for every other node we set

$$\rho_a(n) = Z_a(n) + \max_{\mu \in \mathcal{D}_a(n)} \sum_{m \in n+} \mu(m)\rho_a(m).$$

Dynamic risked competitive equilibrium

Consider a set of agents $a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}$ and stochastic process of inflows for each $a \in \mathcal{H}$ defined by a scenario tree with nodes $n \in \mathcal{N}$ and leaves $\mathcal{L}$. A dynamic risked equilibrium is a stochastic process of energy prices $\{p(n) \mid n \in \mathcal{N}\}$ in the scenario tree, and for each agent a a stochastic process of production/consumption decisions $\{x_a(n) \mid n \in \mathcal{N}\}$, with the property that

$$0 \leq \sum_{a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}} x_a(n) \perp p(n) \geq 0, \; n \in \mathcal{N}$$

and $x_a(n)$ is a solution to the risk-averse optimization problem where agent a at node n minimizes $\rho_a(n)$ evaluated using prices $\{p_n \mid n \in \mathcal{N}\}$ and risk sets $\mathcal{D}_a(n)$, $n \in \mathcal{N} \setminus \mathcal{L}$.

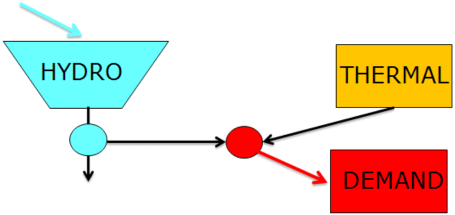
Computing competitive equilibria using EMP
(Ferris, Dirkse, Jagla, Meeraus, 2013)

We use the EMP framework in GAMS. One feature is modeling Multiple Optimization Problems with Equilibrium Constraints (MOPECs). Each agent a in $\mathcal{H} \cup \mathcal{T} \cup \mathcal{C}$ determines her decisions x_a by solving, independently, an optimization problem,

$$x_a \in \mathrm{argmax} f_a(p, x, x_{-a}), \qquad a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C},$$

where x_{-a} are actions of competitors, and $p \in \mathbb{R}^d$ are prices that satisfy a global equilibrium constraint that represents market clearing.

Example: three agents, two periods, 10 inflow scenarios



$$
\begin{aligned}
C(v) &= v^2 \\
U(u) &= 1.5u - 0.015u^2 \\
V(x) &= 10\log(0.1x + 0.01) \\
D(d) &= 40d - 2d^2 \\
D_i &= \mathrm{conv}\{(0.28, 0.08, \ldots, 0.08), (0.08, 0.28, \ldots, 0.08), \ldots, (0.08, 0.08, \ldots, 0.28)\}
\end{aligned}
$$

Example: risk neutral equilibrium

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.336	7.590	6.410	0.668				
1	1	2.539	2.865	5.725	1.269	2.057	20.417	362.283	384.758
1	2	2.053	3.590	6.000	1.027	1.500	19.418	366.863	387.781
1	3	1.696	4.387	6.203	0.848	1.165	18.809	370.264	390.238
1	4	1.431	5.236	6.355	0.716	0.958	18.514	372.809	392.281
1	5	1.231	6.121	6.470	0.616	0.825	18.445	374.746	394.016
1	6	1.076	7.031	6.559	0.538	0.735	18.529	376.252	395.516
1	7	0.953	7.961	6.629	0.477	0.673	18.716	377.446	396.835
1	8	0.855	8.904	6.686	0.427	0.629	18.969	378.411	398.008
1	9	0.774	9.857	6.733	0.387	0.596	19.264	379.204	399.064
1	10	0.706	10.818	6.772	0.353	0.571	19.585	379.866	400.022

Table: Risk neutral equilibrium.

Example: risk averse equilibrium

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.317	7.580	6.420	0.658				
1	1	2.545	2.858	5.722	1.272	2.053	20.280	362.407	384.740
1	2	2.057	3.582	5.998	1.029	1.492	19.277	367.002	387.771
1	3	1.700	4.378	6.202	0.850	1.156	18.664	370.413	390.233
1	4	1.434	5.226	6.353	0.717	0.948	18.366	372.965	392.279
1	5	1.233	6.111	6.469	0.616	0.814	18.295	374.908	394.017
1	6	1.077	7.022	6.558	0.539	0.724	18.378	376.418	395.520
1	7	0.955	7.951	6.629	0.477	0.661	18.564	377.615	396.840
1	8	0.856	8.894	6.686	0.428	0.617	18.816	378.582	398.015
1	9	0.775	9.847	6.733	0.387	0.584	19.111	379.377	399.071
1	10	0.707	10.808	6.772	0.353	0.559	19.432	380.040	400.031

Table: Risk averse equilibrium.

Example: risk averse social plan

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.545	7.710	6.290	0.773				
1	1	2.472	2.948	5.763	1.236	2.125	21.918	360.888	384.931
1	2	2.004	3.682	6.028	1.002	1.601	20.968	365.307	387.876
1	3	1.660	4.486	6.224	0.830	1.286	20.401	368.589	390.276
1	4	1.404	5.340	6.370	0.702	1.090	20.138	371.050	392.277
1	5	1.210	6.229	6.482	0.605	0.963	20.090	372.927	393.980
1	6	1.060	7.142	6.568	0.530	0.878	20.189	374.390	395.457
1	7	0.940	8.073	6.637	0.470	0.818	20.385	375.553	396.756
1	8	0.844	9.018	6.692	0.422	0.775	20.644	376.495	397.914
1	9	0.765	9.972	6.738	0.382	0.743	20.944	377.270	398.957
1	10	0.699	10.934	6.776	0.349	0.719	21.267	377.919	399.905

Table: Risk averse social plan.

Contracts to decrease risk
(Heath and Ku 2004, Ralph and Smeers, 2013)

Suppose we introduce 10 contracts to trade risk, one for each scenario.

We can model a contract to trade risk as an Arrow-Debreu security.

Contract m has a payoff at time 1 of \$1 if scenario m occurs. Agent a buys $Y_a(m)$ (possibly -ve) of these contracts at time 0 at prices $\mu(m)$, and so pays $\sum_m \mu(m) Y_a(m)$.

The market is complete since a portfolio of contracts can be constructed to replicate any set of possible time 1 payoffs. The market for contracts must clear, so

$$\sum_a Y_a(m) = 0.$$

Competitive risk-averse equilibrium

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.545	7.710	6.290	0.773				
1	1	2.472	2.948	5.763	1.236	-1.232	18.320	367.842	384.931
1	2	2.004	3.682	6.028	1.002	-0.039	19.568	368.347	387.876
1	3	1.660	4.486	6.224	0.830	0.700	20.309	369.267	390.276
1	4	1.404	5.340	6.370	0.702	1.405	21.045	369.826	392.277
1	5	1.210	6.229	6.482	0.605	1.999	21.663	370.319	393.980
1	6	1.060	7.142	6.568	0.530	2.510	22.189	370.758	395.457
1	7	0.940	8.073	6.637	0.470	2.956	22.647	371.153	396.756
1	8	0.844	9.018	6.692	0.422	3.353	23.050	371.511	397.914
1	9	0.765	9.972	6.738	0.382	3.708	23.410	371.838	398.957
1	10	0.699	10.934	6.776	0.349	4.031	23.735	372.139	399.905

Table: Risk-averse competitive equilibrium with risk trading.

Trading risk

t	ω_m	price	trade (T)	trade (H)	trade (C)
0			-5.015	-4.366	9.381
1	1	0.280	1.658	0.768	-2.426
1	2	0.080	3.375	2.966	-6.341
1	3	0.080	4.429	4.274	-8.703
1	4	0.080	5.330	5.274	-10.604
1	5	0.080	6.051	5.938	-11.989
1	6	0.080	6.647	6.366	-13.013
1	7	0.080	7.153	6.627	-13.781
1	8	0.080	7.593	6.772	-14.364
1	9	0.080	7.980	6.832	-14.813
1	10	0.080	8.327	6.834	-15.161

Table: Risk trading receipts of the three agents in equilibrium

Trading risk

stage	ω_m	price	trade (T)	trade (H)	trade (C)
0			0	0	0
1	1	0.280	-3.357	-3.598	6.955
1	2	0.080	-1.640	-1.400	3.040
1	3	0.080	-0.586	-0.092	0.678
1	4	0.080	0.315	0.908	-1.223
1	5	0.080	1.036	1.573	-2.609
1	6	0.080	1.632	2.000	-3.632
1	7	0.080	2.138	2.262	-4.400
1	8	0.080	2.578	2.406	-4.983
1	9	0.080	2.965	2.467	-5.432
1	10	0.080	3.312	2.468	-5.780

Table: Net receipts of the three agents in equilibrium

Competitive risk-averse social plan

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.545	7.710	6.290	0.773				
1	1	2.472	2.948	5.763	1.236	2.125	21.918	360.888	384.931
1	2	2.004	3.682	6.028	1.002	1.601	20.968	365.307	387.876
1	3	1.660	4.486	6.224	0.830	1.286	20.401	368.589	390.276
1	4	1.404	5.340	6.370	0.702	1.090	20.138	371.050	392.277
1	5	1.210	6.229	6.482	0.605	0.963	20.090	372.927	393.980
1	6	1.060	7.142	6.568	0.530	0.878	20.189	374.390	395.457
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1	9	0.765	9.972	6.738	0.382	0.743	20.944	377.270	398.957
1	10	0.699	10.934	6.776	0.349	0.719	21.267	377.919	399.905

Table: Risk-averse social planning solution using intersection of risk sets. Adding receipts from risk trading give risked equilibrium.

Competitive risk-averse equilibrium

t	ω_m	price	storage	release	thermal	profit (T)	profit (H)	welfare (C)	welfare (total)
0		1.545	7.710	6.290	0.773				
1	1	2.472	2.948	5.763	1.236	-1.232	18.320	367.842	384.931
1	2	2.004	3.682	6.028	1.002	-0.039	19.568	368.347	387.876
1	3	1.660	4.486	6.224	0.830	0.700	20.309	369.267	390.276
1	4	1.404	5.340	6.370	0.702	1.405	21.045	369.826	392.277
1	5	1.210	6.229	6.482	0.605	1.999	21.663	370.319	393.980
1	6	1.060	7.142	6.568	0.530	2.510	22.189	370.758	395.457
1	7	0.940	8.073	6.637	0.470	2.956	22.647	371.153	396.756
1	8	0.844	9.018	6.692	0.422	3.353	23.050	371.511	397.914
1	9	0.765	9.972	6.738	0.382	3.708	23.410	371.838	398.957
1	10	0.699	10.934	6.776	0.349	4.031	23.735	372.139	399.905

Table: Risk-averse competitive equilibrium with risk trading.

Risk trading in a multi-stage setting

In each node $n \in \mathcal{N} \setminus \mathcal{L}$ agents might trade the risk of future positions. This involves a contract that pays $\sum_{m \in n+} \mu(m) Y_a(m)$ in node $n \in \mathcal{N}$, to receive payments of $Y_a(m)$ in node $m \in n+$. Suppose there is a complete market for risk in each $n \in \mathcal{N} \setminus \mathcal{L}$ (i.e. contracts traded in node n span the $|n+|$ payoff outcomes). Let

$$\mathcal{D}_0(n) = \cap_{a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}} \mathcal{D}_a(n) \neq \emptyset,$$

and let $\{x_a(n) \mid n \in \mathcal{N}, a \in \mathcal{H} \cup \mathcal{T} \cup \mathcal{C}\}$ be a solution to the risk-averse social planning problem with risk sets $\mathcal{D}_0(n)$. Suppose this gives shadow prices $\{p(n) \mid n \in \mathcal{N}\}$. These prices and quantities form a dynamic risked equilibrium in which agents trade risk i.e. agent a at node n minimizes $\rho_a(n)$ with a policy defined by $x_a(\cdot)$ together with a policy of risk trading at node n and its children.

Conclusions

- If the social planner (e.g. a market regulator or a single-buyer optimizer) has knowledge of agent's risk sets, and complete risk markets, then the planner can compute a competitive risk-averse equilibrium.
- In practice, this will be diffcult: risk measures are private information, and markets for risk in hydro inflows will be incomplete.
- Incomplete markets: equilibrium solutions are computable for small problems, but a challenge to scale to multi-stage problems (an open problem).

Implications for market regulators

- Attention in electricity markets focuses on possible exercise of unilateral market power.
- Modelling strategic behaviour in hydro-dominated systems is very challenging (e.g. Scott and Read 1996, Bushnell 2003, Rangel 2008).
- Modelling perfectly competitive behaviour in complete markets (via optimization) is easier.
- Enables focus on the effects of uncertainty and market incompleteness on competitive outcomes.
- Compare with observed market outcomes and try to identify market design improvement e.g. hedging instruments.

This is the end

THE END